Statistical Quality Control

Module 6

July 16, 2014

Overview
Statistical Quality Control Systems

- What is a sample?
- Basic forms of sampling
  - Statistical Process Control (SPC)
- Control charts
  - What are they
  - Interpreting control charts
  - Constructing control charts
  - Examples

Two Scoops of Raisins in a Box of Kellogg's Raisin Bran

- 1. Cereal production → Highly Automated Process
- 2. Kellogg's Uses Quality Management Techniques Such as Establishing Conformance Standards, Sampling, and Statistical Process Control

Two Scoops of Raisins in a Box of Kellogg’s Raisin Bran

- 3. Statistical Quality Control Charts are Used to Determine Whether the Variations Observed from One Cereal Box to the Next are Random or Have a Specific Cause

- 4. Quality Insurance Inspectors Periodically Open Random Samples of the Packed Boxes that are Ready to be Shipped

Statistical Quality Control

- 1. The use of statistical tools and analysis to control and improve the quality of a product or service.
- 2. One of the sets of tools for Total Quality Management (TQM)
- 3. Central to the strategies promoted by the pioneers of the quality movement, such as Deming, Juran, and Taguchi
- 4. “If you can’t describe and measure it, then you can’t control or improve it.”

Illustrations

- 1. BASF – catalytic cores for pollution control
  - A. Percentage of defective cores
- 2. Milliken – industrial fabrics
  - A. Number of defects per 100 yards
- 3. Thermalex – thermal tubing
  - A. Height and width (variables)
- 4. Land’s End – customer service, order fulfillment
  - A. Percentage of correctly filled orders
- 5. Hospital pharmacy
  - A. Prescription error rate

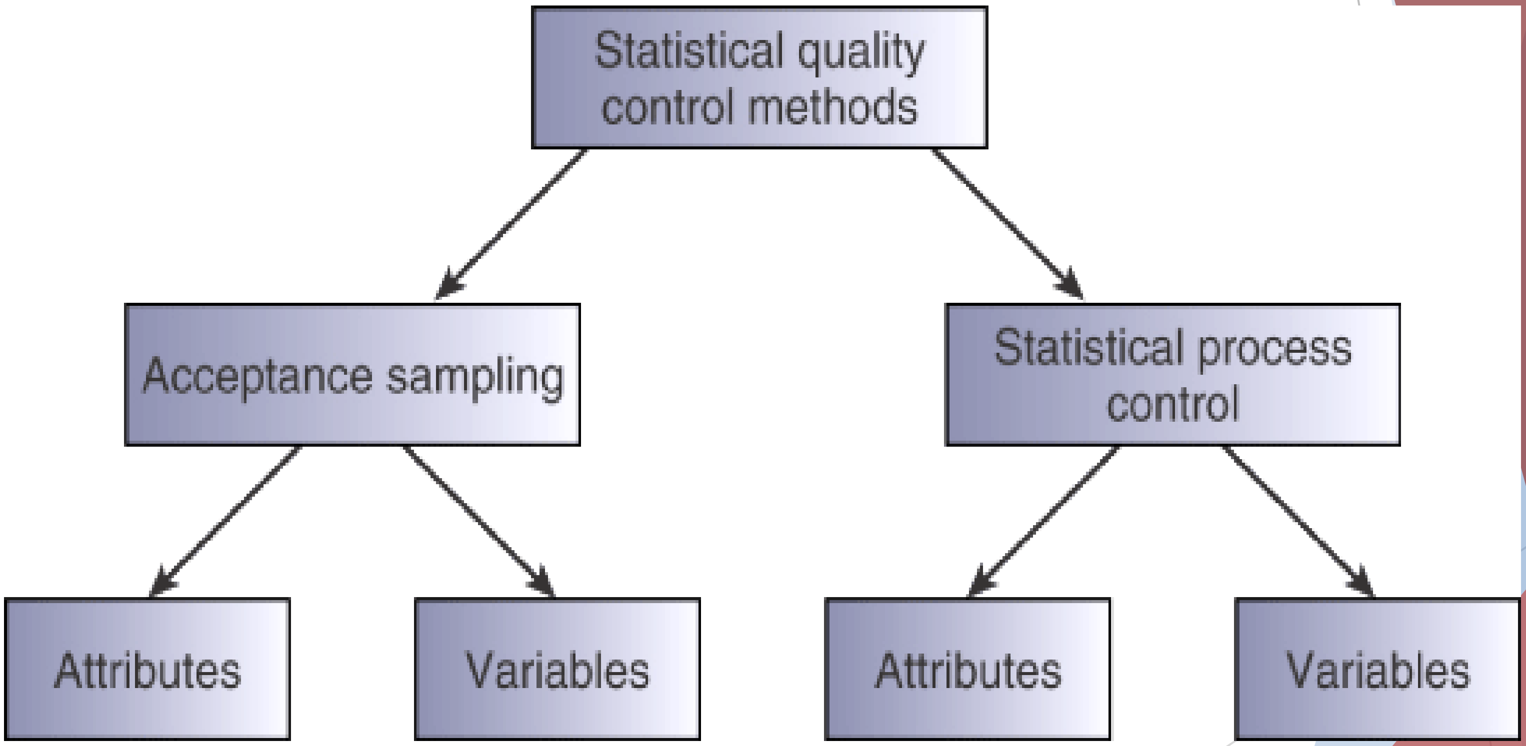
Steps in Designing Statistical Quality Control Systems

1. Identify critical points
   - Incoming materials & services
   - Work in process
   - Finished product or service
2. Decide on the type of measurement
   - variables
   - attribute
3. Decide on the amount of inspection to be used.
4. Decide who should do the inspection—end of line inspectors, or workers inspecting their own work?

When Someone Finds a Defect…

1. Containment: Keep the defective items from getting to the customer
2. Correction: Find the cause of the defect and correct it.
3. Prevention: Prevent the cause from happening again.
4. Continuously improve the system.

Statistical Quality Control Methods



Basic Forms of Statistical Sampling

- Statistical process control (SPC)
  - Sampling to determine if the process is within acceptable limits
- Acceptance sampling
  - Sampling to accept or reject the immediate lot of *product* at hand

Variation

- 1. It always exists!
- 2. Processes and products never turn out exactly the same.
- 3. Goals
  - A. Measure the variation
  - B. Understand the causes of variation
  - C. Reduce the variation

Sources of Variation

- 1. Common causes of variation
  - A. Random causes that we cannot identify
  - B. Unavoidable
  - C. e.g. slight differences in process variables like diameter, weight, service time, temperature
  - D. Deming Funnel: http://www.symphonytech.com/dfunnel.htm
- 2. Assignable causes of variation
  - A. Causes can be identified and eliminated
  - B. e.g. poor employee training, worn tool, machine needing repair

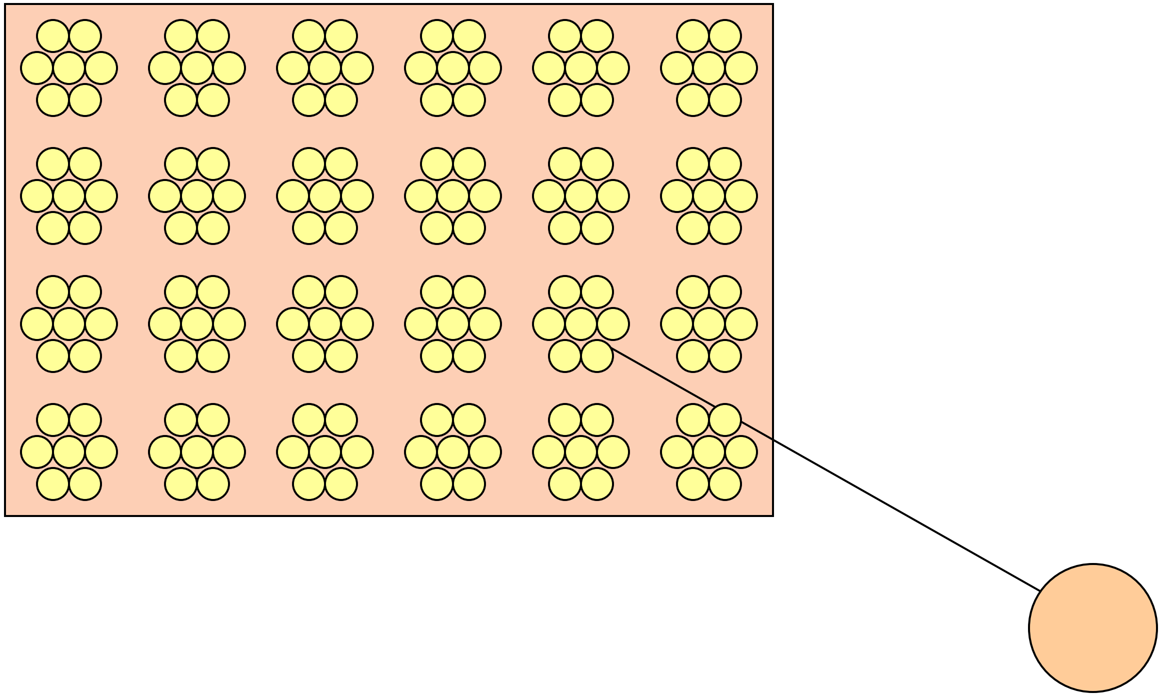
Use Control Charts

A managerial tool used to analyze whether a process is “in control” or “out of control”

graphs that visually show if a sample is within statistical control limits

Sample

Population of interest **(e.g., cars produced, customers served,…)**



Sample: **Subset of the population**

We'll use statistics to judge the quality of the population (lot) based on the quality of the sample

Issues in Using Control Charts

- Sample Size
  - large enough to detect defectives
  - variables can use smaller sample sizes
- How often to sample?
  - Depends upon cost
- Control limits vs. product specifications
  - Is the process capable of producing to specs?
  - Are the specifications appropriate?

Issues in Using Control Charts

- Sample Size
  - large enough to detect defectives
    - E.g. if expect one defect per hundred, sample size should be at least 100.
  - variables can use smaller sample sizes
    - Because computing (e.g. mean) instead of counting.

Issues in Using Control Charts

- How often to sample?
  - Depends upon cost
    - What is the cost of the actual sample?
      - E.g. crash tests for cars
    - What is the cost of the wrong decision?
      - E.g. what happens if you decide a bad batch of medicine is good? (This could happen because of randomness.)

Issues in Using Control Charts

- Control limits vs. product specifications
  - Is the process capable of producing to specs?
    - If not, then change the process or change the specifications.
  - Are the specifications appropriate?
    - How long do you expect the product to last, or under what conditions do you expect it to survive?
      - Should your iPod survive being run over by your car?

Sampling Errors

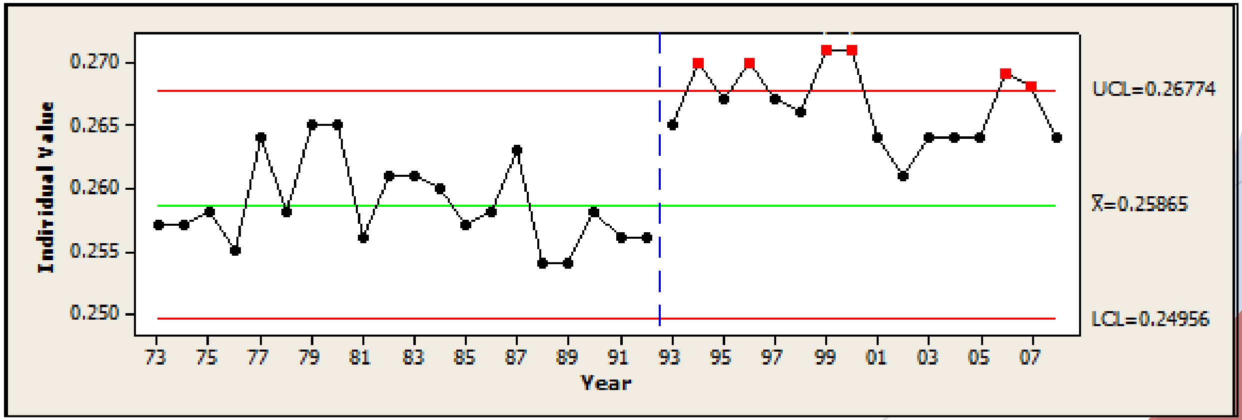
- Type I (α Error or Producer's Risk)
  - Occurs when a sample says parts are bad or the process is out of control when the opposite is true.
  - The probability of rejecting good parts as scrap.
- Type II (β error or Consumer's Risk)
  - Occurs when a sample says parts are good or the process is in control when the reverse is true.
  - The probability of a customer getting a bad lot represented as good.

Types of Sampling Errors

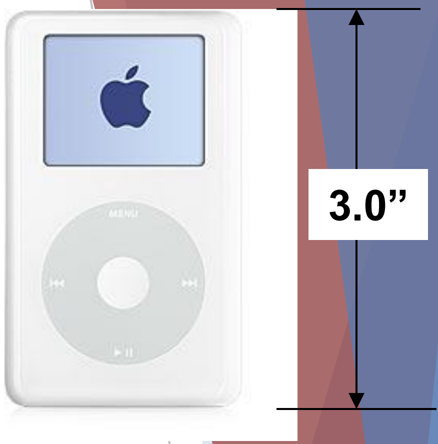
		The population or process is actually:	
		Good or in control	Bad or out of control
The sample says that the population or process is:	Good or in control	In agreement	β or Type II error
	Bad or out of control	α or Type I error	In agreement

SPC Methods-Control Charts

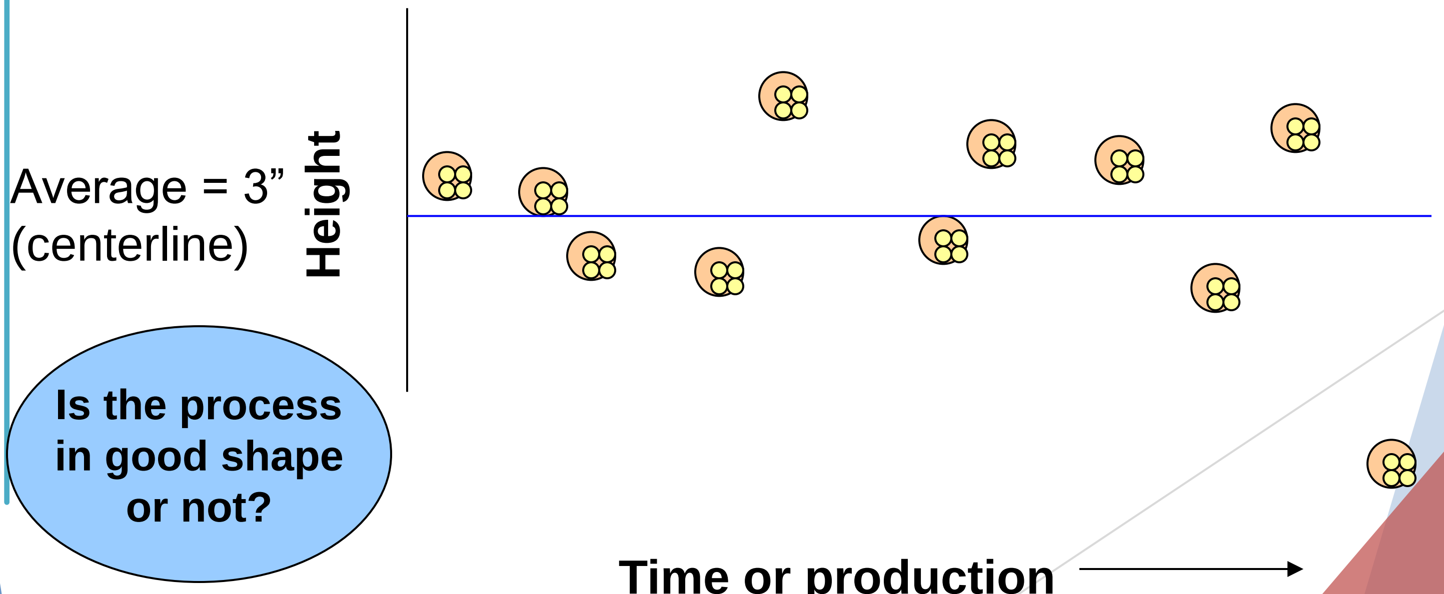
- **1. Control Charts** show sample data plotted on a graph
  - A. Center Line (CL): central tendency of the sample data (mean, average)
  - B. Upper Control Limit (UCL): upper limit for sample data
  - C. Lower Control Limit (LCL): lower limit for sample data



Control Charts



- Suppose we produce ipods
- The average height of the ipod is 3”
- Throughout the day we randomly sample ipods from the production line and measure their heights



Control Charts

In order to answer this we need to judge whether the process is in control or out of control…

- In Control
  - process variation is due to chance or sampling error.
  - variation is within the limits of the normal curve.
  - the process needs no adjustment.

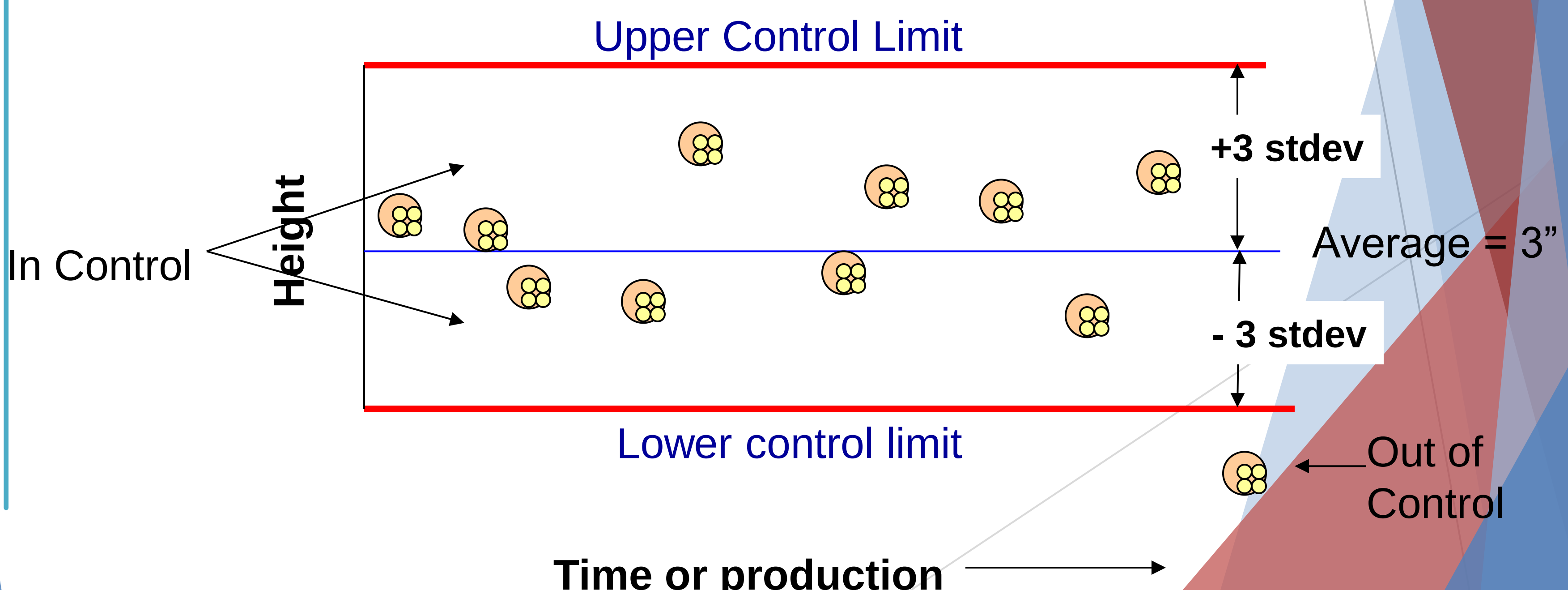
- Out of Control
  - process variation due to some assignable cause.
  - variation is outside limits of the normal curve.
  - the process needs attention or adjustment.

Control Charts

- One way to do this is by adding control limits to our control charts
  - Upper control limit: + 3 standard dev
  - Lower control limit: - 3 standard dev

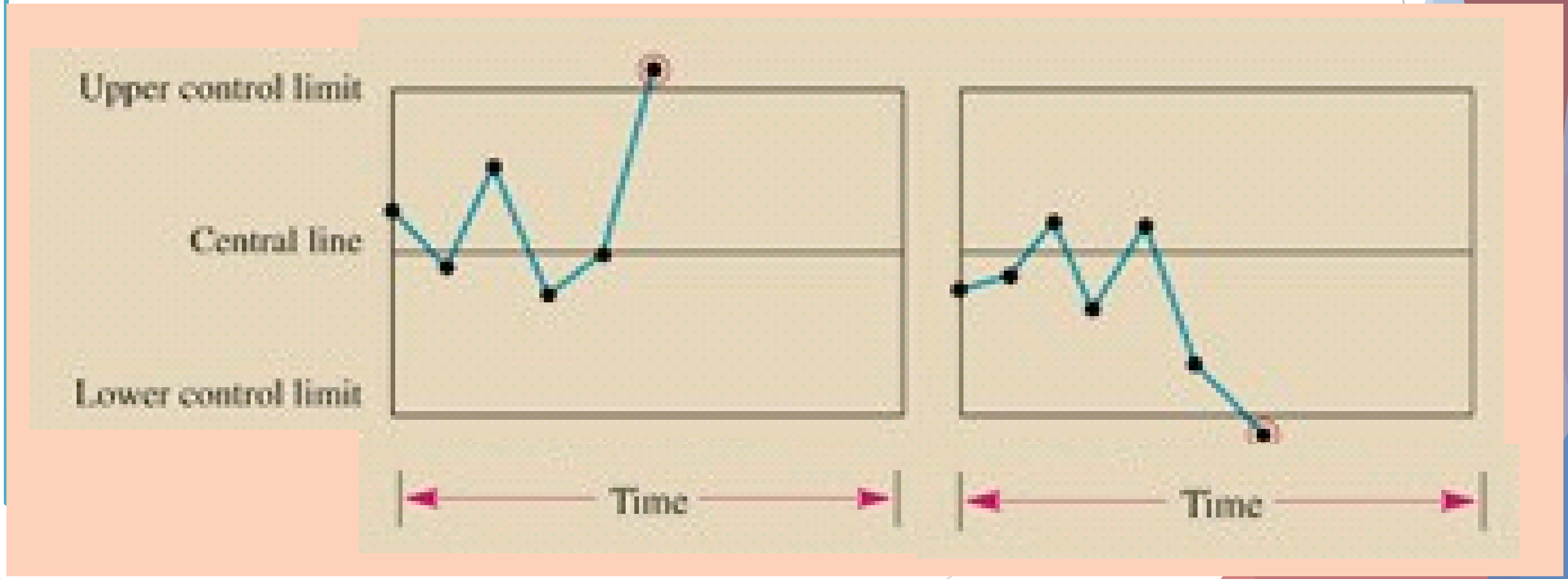
Control Charts

- Suppose we produce ipods
- The average height of the ipod is 3”
- Throughout the day we randomly pull ipods from the production line and measure their height



Interpreting Control Charts – Out of Control

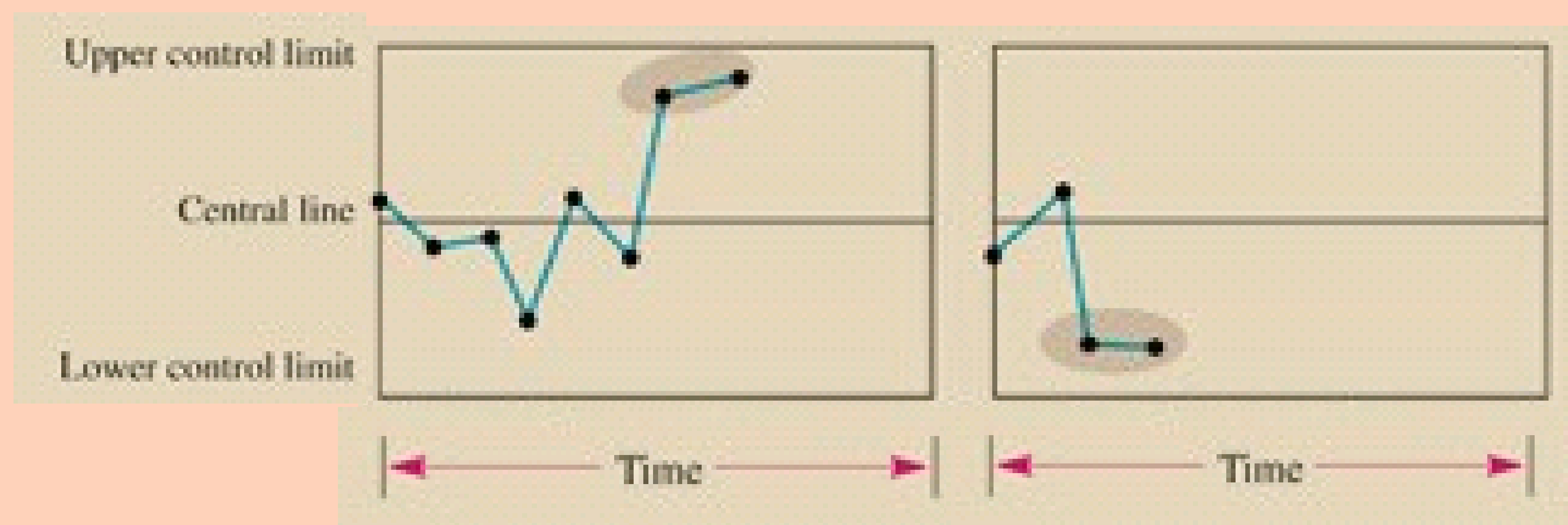
- What might lead us to conclude the system is out of control?
  i. 1 sample statistic outside the control limits



Interpreting Control Charts – Out of Control

- What might lead us to conclude the system is out of control?

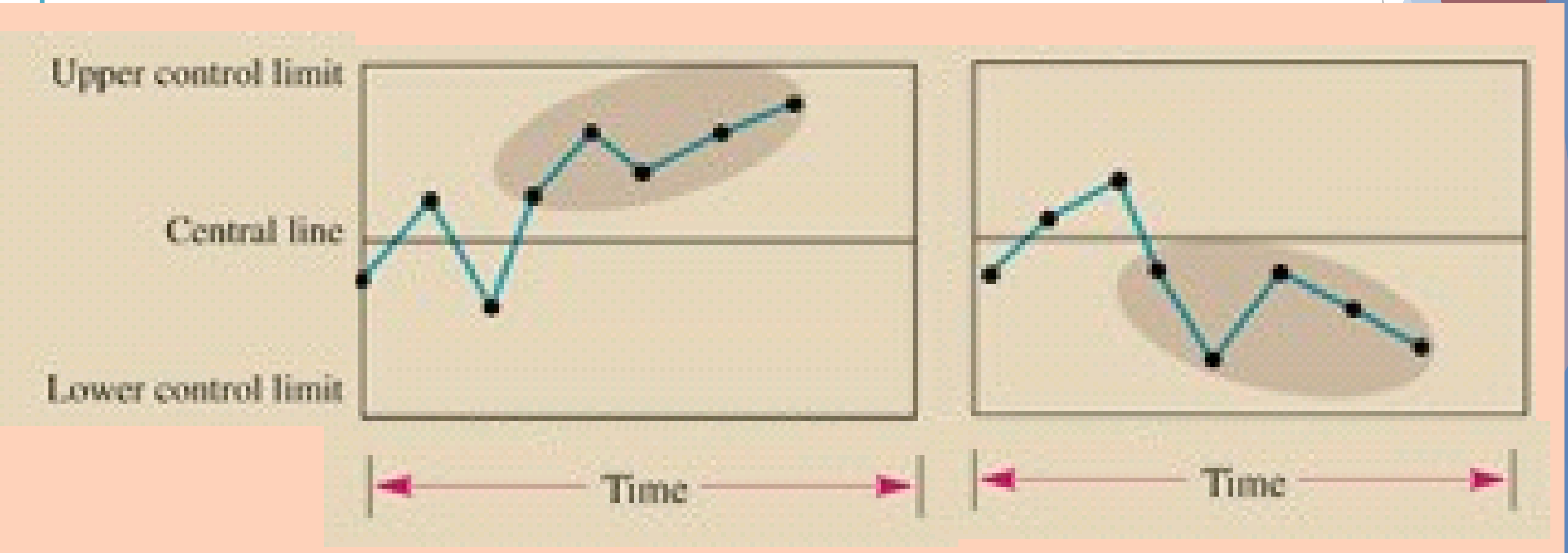
    ii. 2 consecutive sample statistics near the control limits



Interpreting Control Charts – Out of Control

- What might lead us to conclude the system is out of control?

  iii. 5 consecutive points above or below the central line

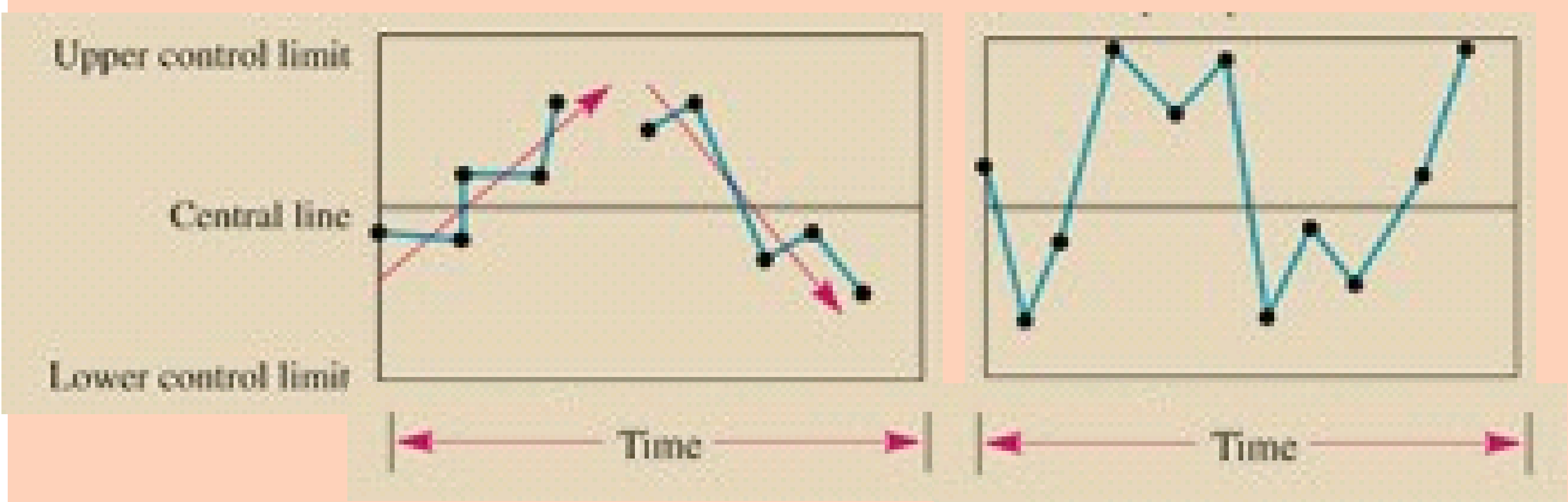


Interpreting Control Charts – Out of Control

- What might lead us to conclude the system is out of control?

  iv. A trend of 5 consecutive points

  v. Very erratic behavior



Types Of Measurement
Attribute measurement

- Attributes are counts, such as the number (or proportion) of defects in a sample.

- Product characteristic evaluated with a discrete choice: Good/bad, yes/no

- .

Types Of Measurement
Variables measurement

- Variables are measures (mean & range or standard deviation) of critical characteristics in a sample.
  - Product characteristic that can be measured on a continuous scale:
    - Length, size, weight, height, time, velocity

Control Charts for Attributes

- *p*-Charts
  - 1. Calculate the proportion of defective parts in each sample
  - 2. Use <u>P-Charts</u> for quality characteristics that are discrete and involve <u>yes/no or good/bad</u> decisions
  - 3. Number of leaking caulking tubes in a box of 48
  - 4. Number of broken eggs in a carton

P-Chart Example

Sample	Number of Defective Tires	Number of Tires in each Sample	Proportion Defective
1	3	20	
2	2	20	
3	1	20	
4	2	20	
5	1	20	
Total	9	100	

- A Production manager for a tire company has inspected the number of defective tires in five random samples with 20 tires in each sample. The table shows the number of defective tires in each sample of 20 tires. Calculate the proportion defective for each sample, the center line, and control limits using $z = 3.00$.

P-Chart Example, cont.

$$n = 20,\ z = 3.00$$

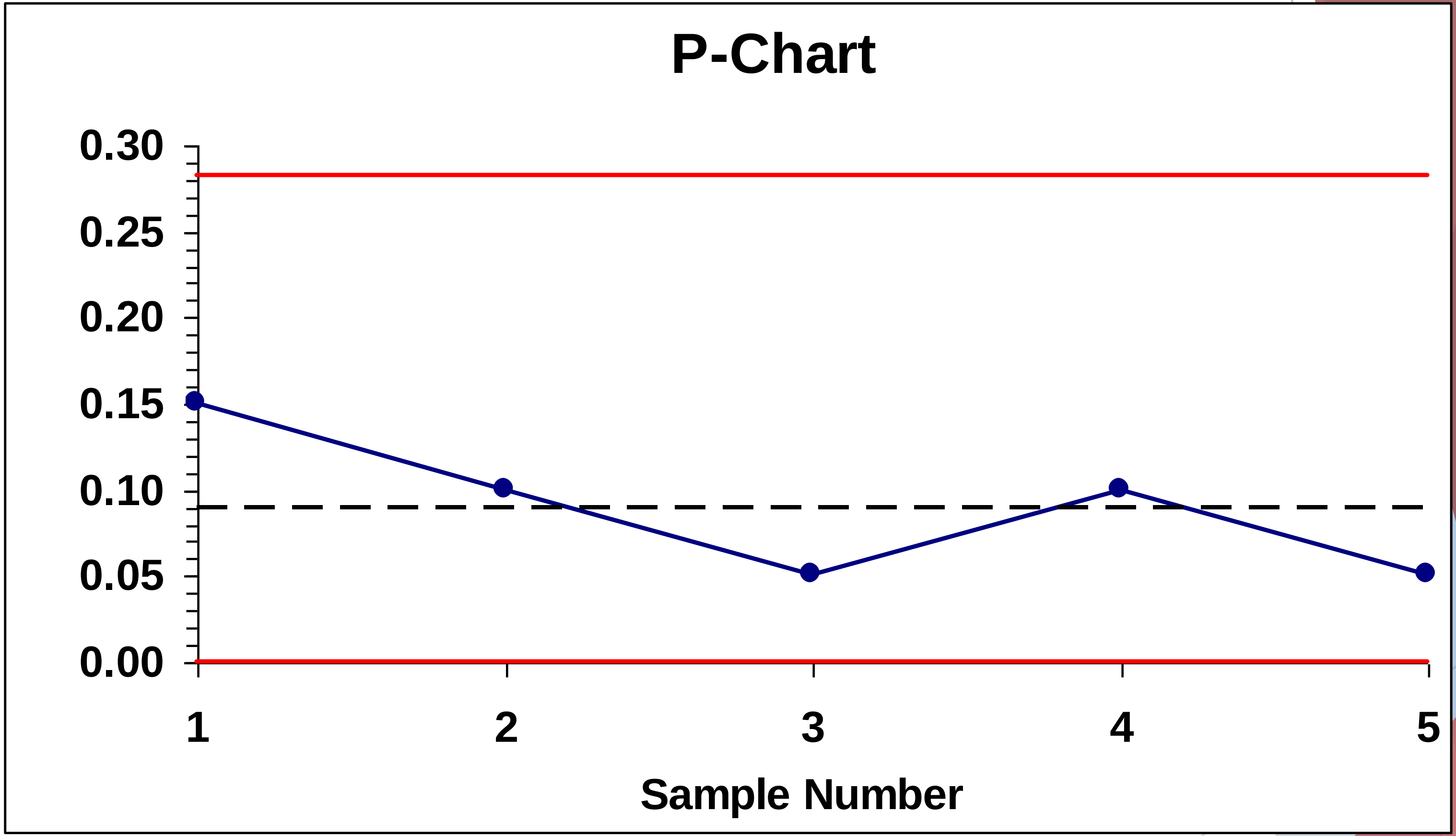
$$\text{CL} = \overline{p} = \frac{\#\text{Defectives}}{\text{Total Inspected}} = \frac{9}{100} = 0.09$$

$$\sigma_p = \sqrt{\frac{(0.09)(0.91)}{20}} = 0.064$$

$$UCL = 0.09 + 3(0.064) = 0.282$$

$$LCL = 0.09 - 3(0.064) = -0.102 \Rightarrow 0$$

P-Chart Example, cont.



Control Charts for Attributes

- *c*-Charts
  - 1. Count the number of defects found in each sample or observation period (possibly more than one defect per part)
  - 2. Use C-Charts for discrete defects when there can be more than one defect per unit
  - 3. Number of flaws or stains in a carpet sample cut from a production run
  - 4. Number of complaints per customer at a hotel

C-Chart Calculations

$\bar{c} =$ average number of defects per sample or observation period

$\sigma_c = \sqrt{\bar{c}} =$ std. deviation of percent defective in a sample

$z =$ number of std. deviations away from process average

(usually 3.0 or 2.0)

$$UCL = \bar{c} + z\sigma_c$$

$$CL = \bar{c}$$

$$LCL = \bar{c} - z\sigma_c \geq 0$$

C-Chart Example

- The number of weekly customer complaints are monitored in a large hotel using a c-chart. Develop three sigma control limits using the data table below.

Week	Number of Complaints
1	3
2	2
3	3
4	1
5	3
6	3
7	2
8	1
9	3
10	1
Total	22

C-Chart Example, cont.

$$z = 3.00$$

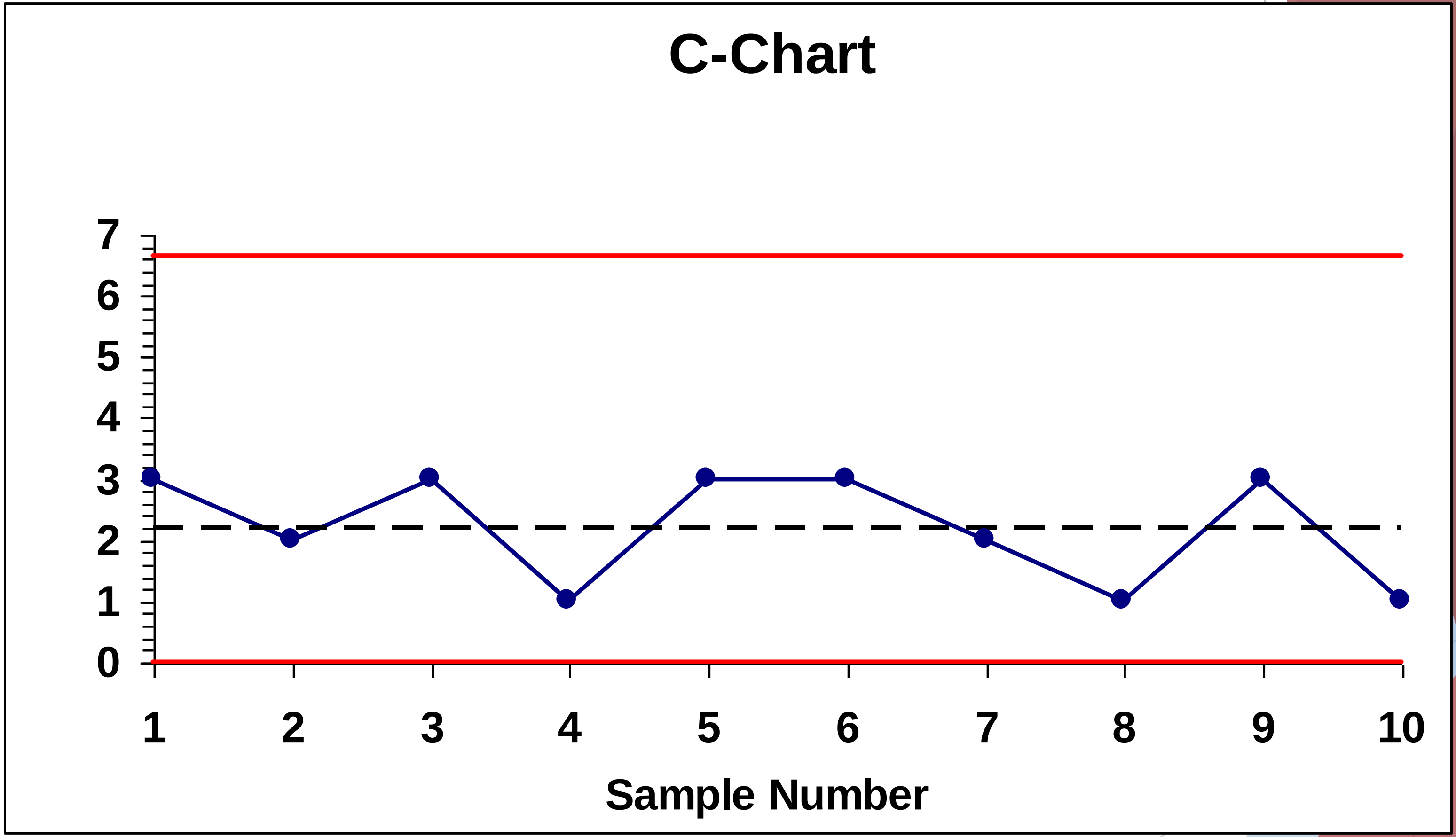
$$\text{CL} = \bar{c} = \frac{\#\text{Complaints}}{\#\ \text{Samples}} = \frac{22}{10} = 2.20$$

$$\sigma_c = \sqrt{2.20} = 1.483$$

$$UCL = 2.20 + 3(1.483) = 6.65$$

$$LCL = 2.20 - 3(1.483) = -2.25 \Rightarrow 0$$

C-Chart Example, cont.



Control Charts for Variables

- **1. Control chart for <u>variables</u>** are used to monitor characteristics that can be measured, such as length, weight, diameter, time
- 2. *X*-bar Chart: Mean
  - A. Plots sample averages
  - B. Measures central tendency (location) of the process
- *3. R* Chart: Range
  - A. Plots sample ranges
  - B. Measures dispersion (variation) of the process
- 4. MUST use BOTH charts together to effectively monitor and control variable quality charateristics

R-Chart Calculations

$n =$ sample size

$k =$ number of samples

$\overline{R} =$ average sample range

Sample Size (n)	Factor for x-Chart A2	Factors for R-Chart D3	D4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

$D_3 =$ LCL parameter

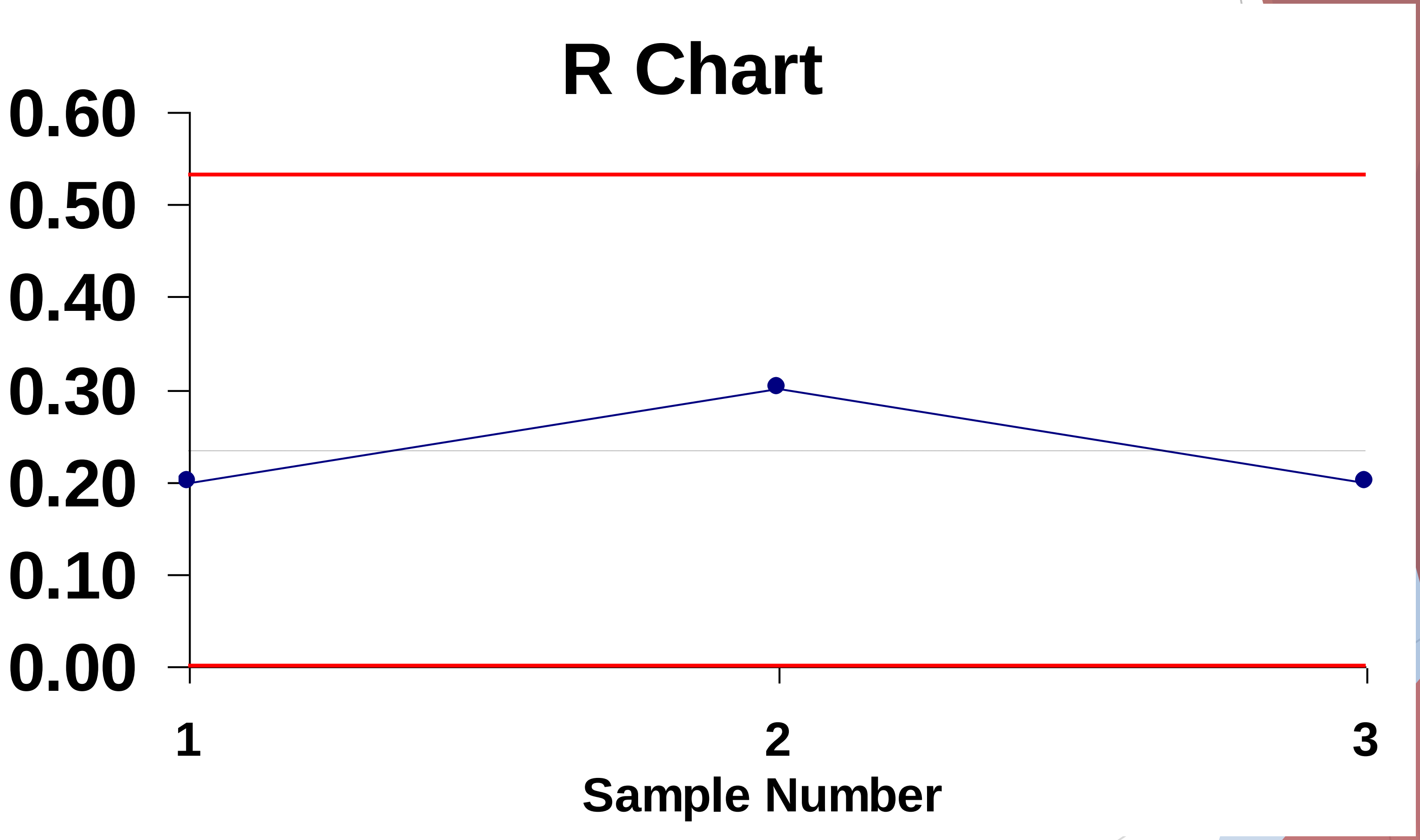
$D_4 =$ UCL parameter

$$UCL = D_4\overline{R}$$

$$CL = \overline{R}$$

$$LCL = D_3\overline{R}$$

R-Chart Example, cont.



X-bar Chart Calculations

$n =$ sample size

$k =$ number of samples

$\overline{\overline{X}} =$ average of the sample means

$A_2 =$ X-bar parameter

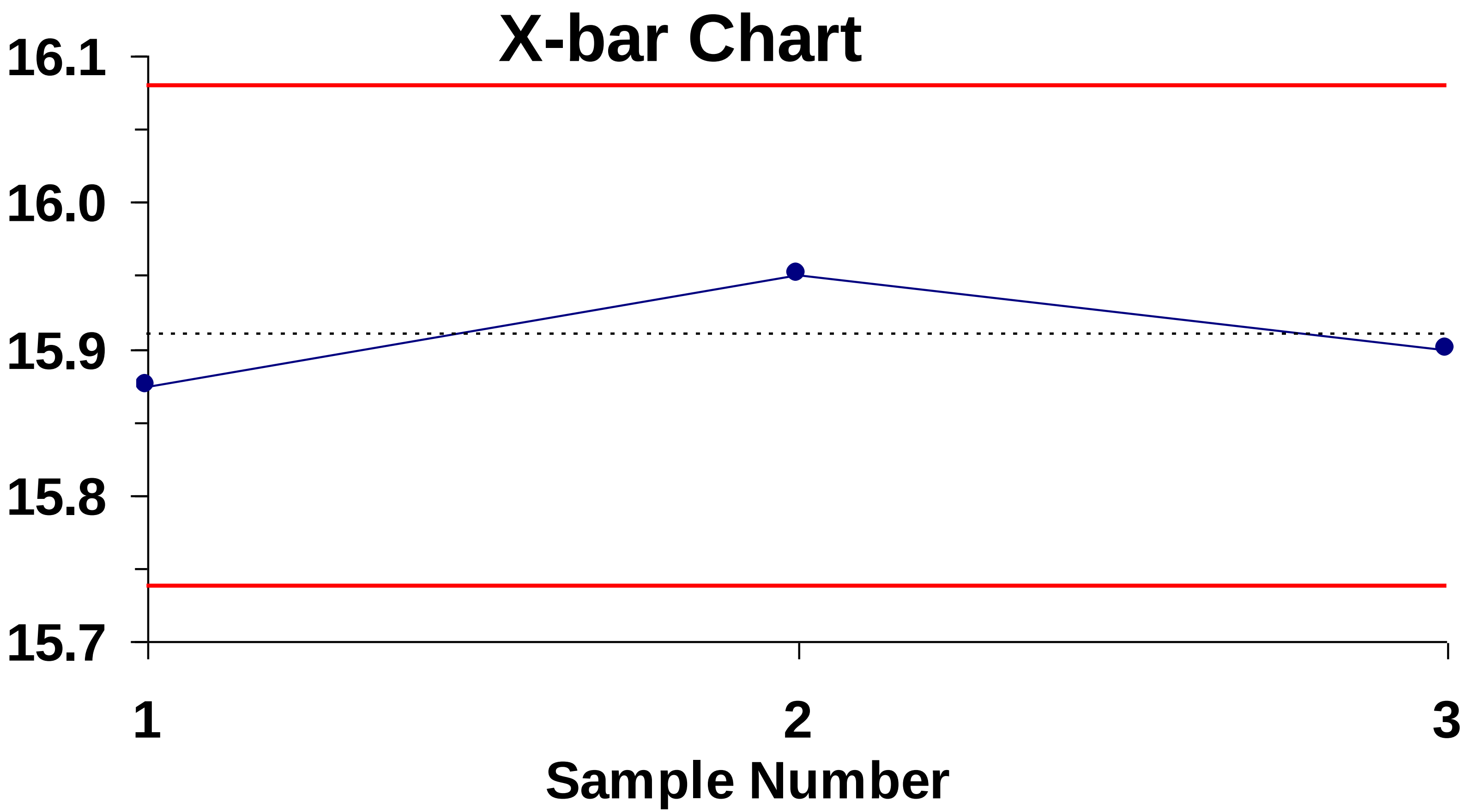
$$UCL = \overline{\overline{X}} + A_2\overline{R}$$

$$CL = \overline{\overline{X}}$$

$$LCL = \overline{\overline{X}} - A_2\overline{R}$$

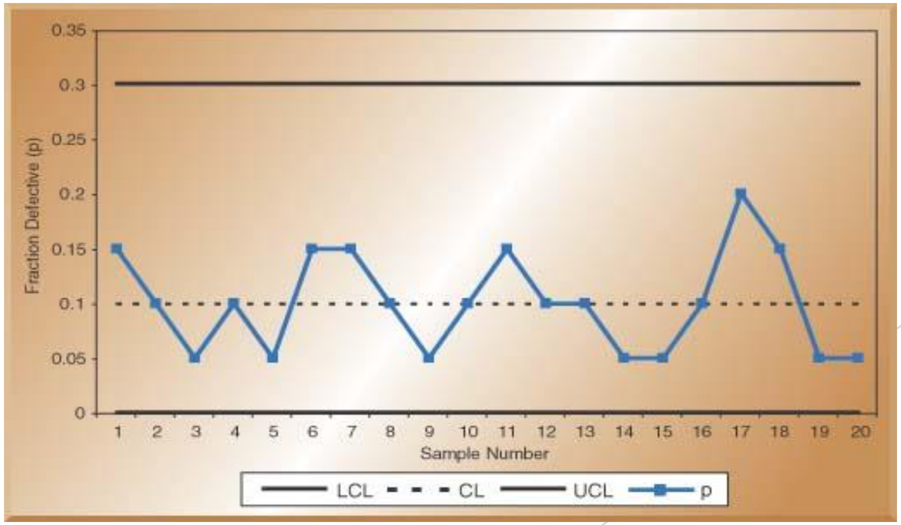
Sample Size (n)	Factor for x-Chart A2	Factors for R-Chart D3	D4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

X-bar Chart Example, cont.



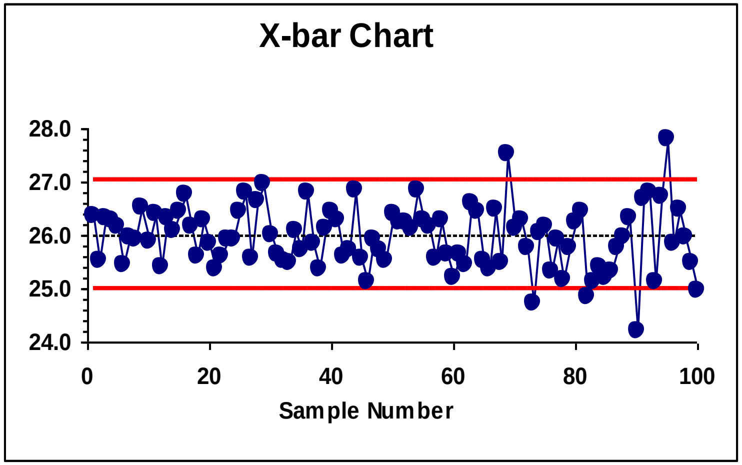
Interpreting Control Charts

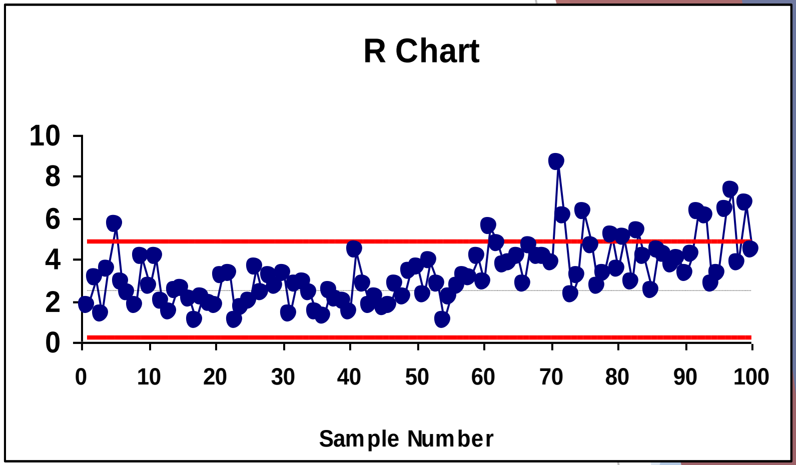
- A process is “in control” if all of the following conditions are met.
  1. No sample points are outside limits
  2. Most sample points are near the process average
  3. About an equal number of sample points are above and below the average
  4. Sample points appear to be randomly distributed



Control Chart Examples

1





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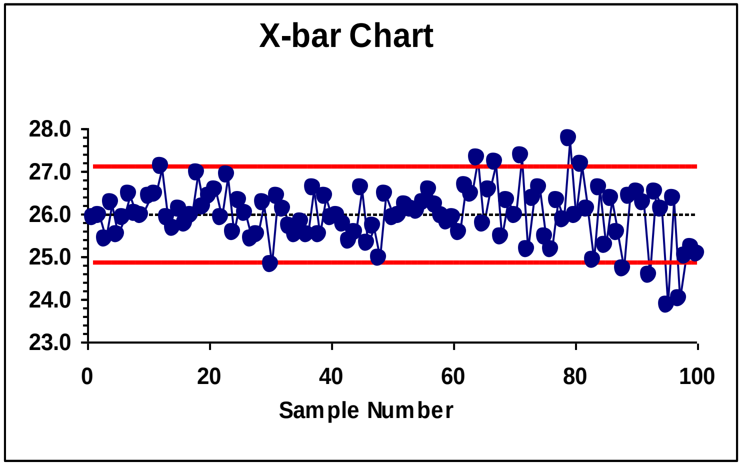
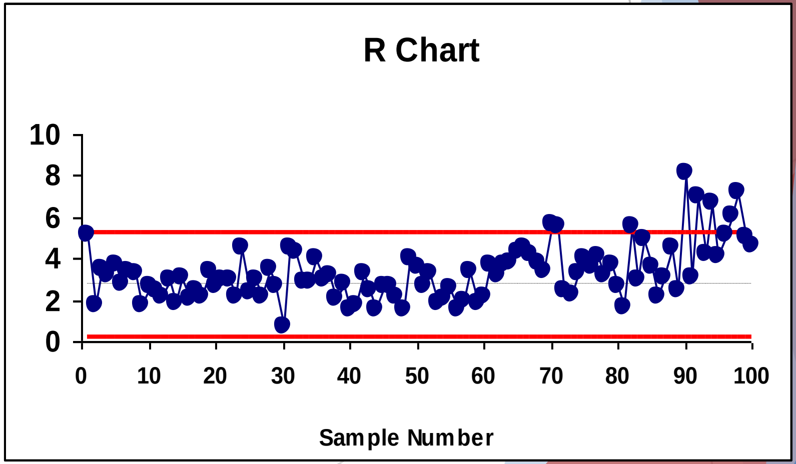
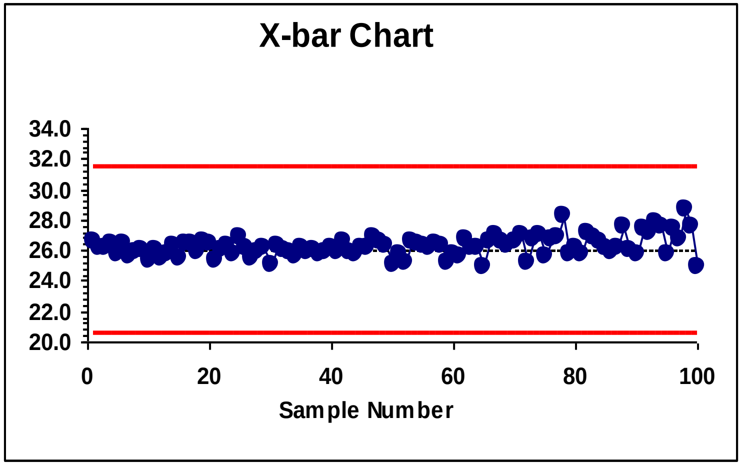
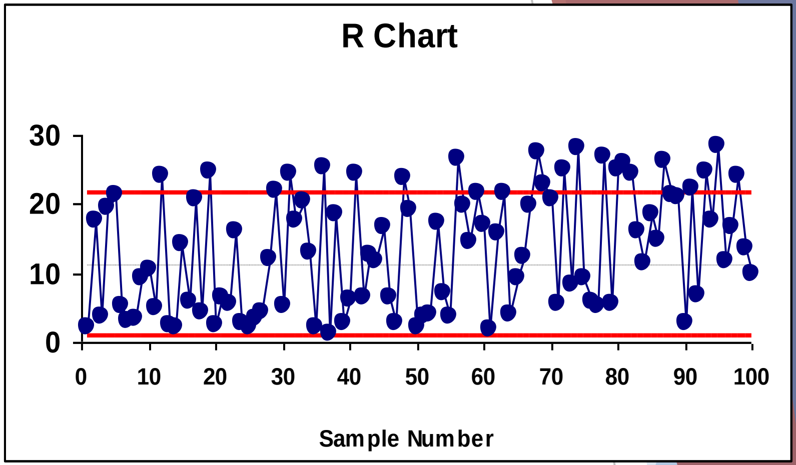




Limits Based on Out of Control Data

3





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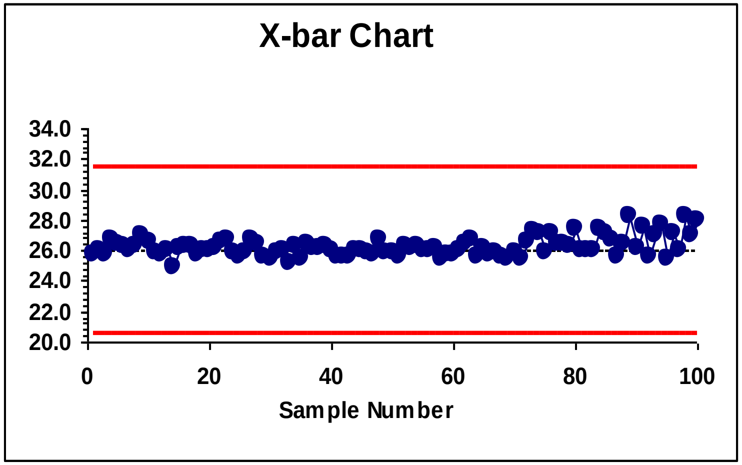
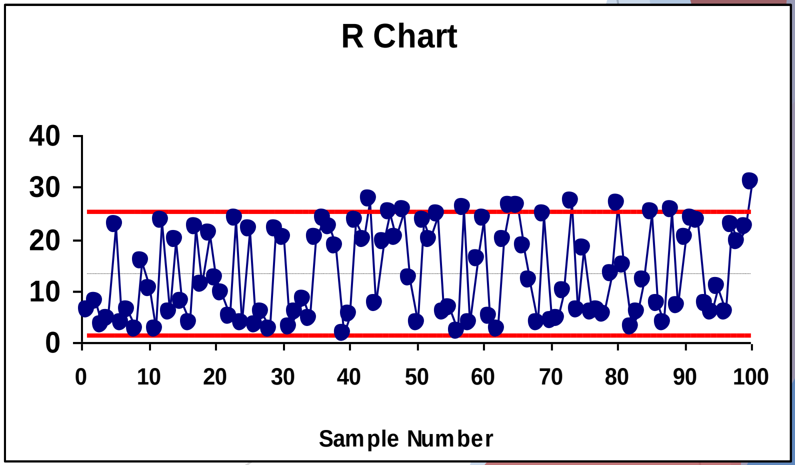




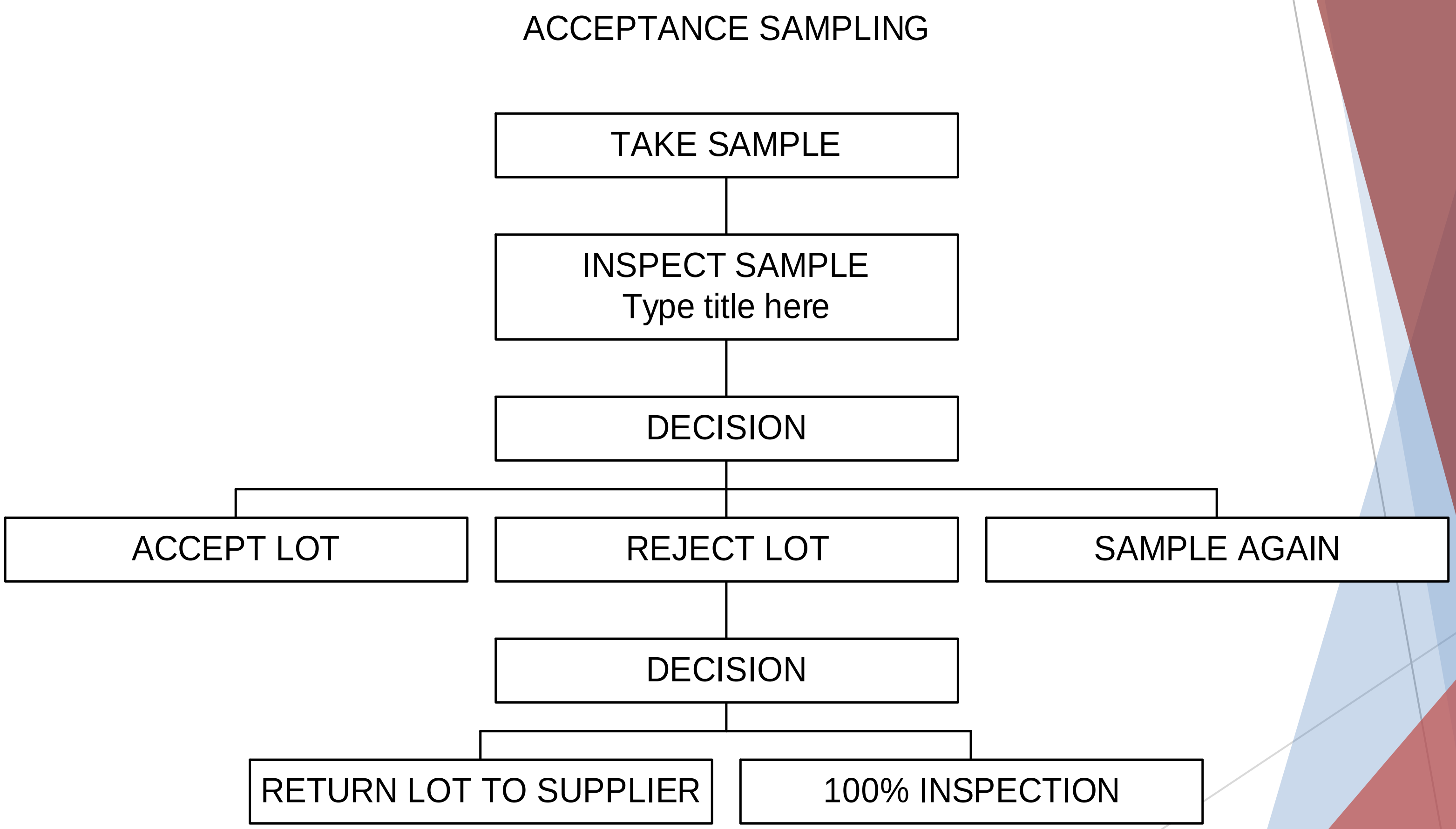
What is acceptance sampling?

- 1. Purposes
  - A. Determine the quality level of an incoming shipment or, at the end production
  - B. Ensure that the quality level is within the level that has been predetermined

What is acceptance sampling?

- 1. Can be either 100% inspection, or a few items of a lot.
- 2. Complete inspection
  - A. Inspecting each item produced to see if each item meets the level desired
  - B. Used when defective items would be very detrimental in some way

Acceptance Sampling Method



What is acceptance sampling?

- 1. Problems with 100% inspection
  - A. Very expensive
  - B. When product must be destroyed to test
  - C. Inspection must be very tedious so defective items do not slip through inspection

Acceptance Sampling – Advantages

- 1. Advantages
  - A. Less handling damages
  - B. Fewer inspectors to put on payroll
  - C. 100% inspection costs are to high
  - D. 100% testing would take to long

Acceptance Sampling – Disadvantages

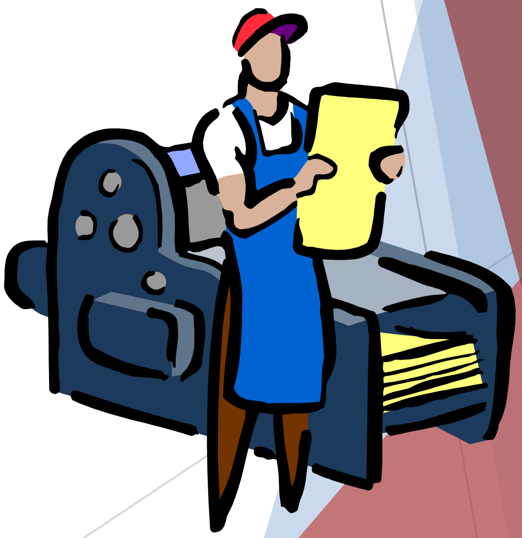
- 1. Disadvantages
  - A. Risk included in chance of bad lot “acceptance” and good lot “rejection”
  - B. Sample taken provides less information than 100% inspection

When is Acceptance Sampling Useful?

- 1. When product testing is:
  - A. destructive
  - B. expensive
  - C. time consuming
- 2. When developing new products
- 3. When dealing with new suppliers
- 4. When a supplier's product hasn't had excellent quality in the past

Risks of Acceptance Sampling

- 1. Producers Risk
  - A. The risk associated with a producer rejecting a lot of materials that actually have good quality
    - a. Also referred to as a Type I Error



Risks of Acceptance Sampling

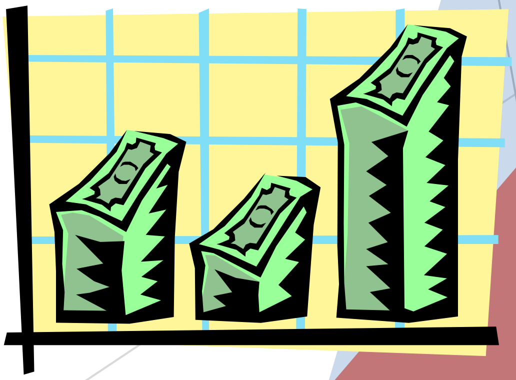
- 1. Consumers Risk
  - A. The risk associated with a consumer accepting a lot of materials that actually have poor quality
    - a. Also referred to as a Type II Error

When can acceptance sampling be applied?

- 1. At any point in production
- 2. The output of one stage is the input of the next

When can acceptance sampling be applied?

- 1. Sampling at the Input stage
  - A. Prevents goods that don’t meet standards from entering into the process
  - B. This saves rework time and money



When can acceptance sampling be applied?

- 1. Sampling at the Output stage
  - A. Can reduce the risk of bad quality being passed on from the process to a consumer
  - B. This can prevent the loss of prestige, customers, and money

When can acceptance sampling be applied?

- 1. Sampling at the Process stage
  - A. Can help adjust the process and reduce the amount of poor quality in production
  - B. Helps to determine the source of bad production and enables return for reprocessing before any further costs may be incurred

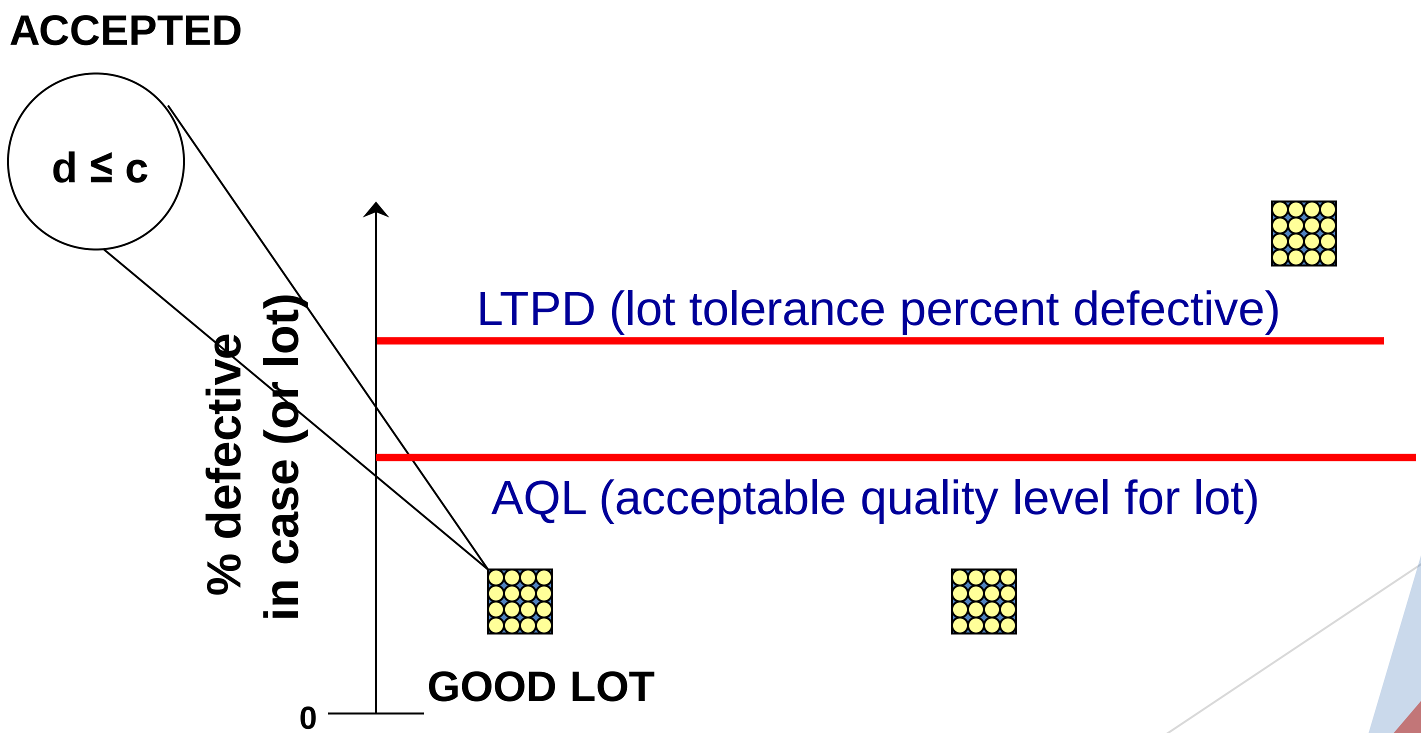
Acceptance Sampling

Inspecting Cookies…

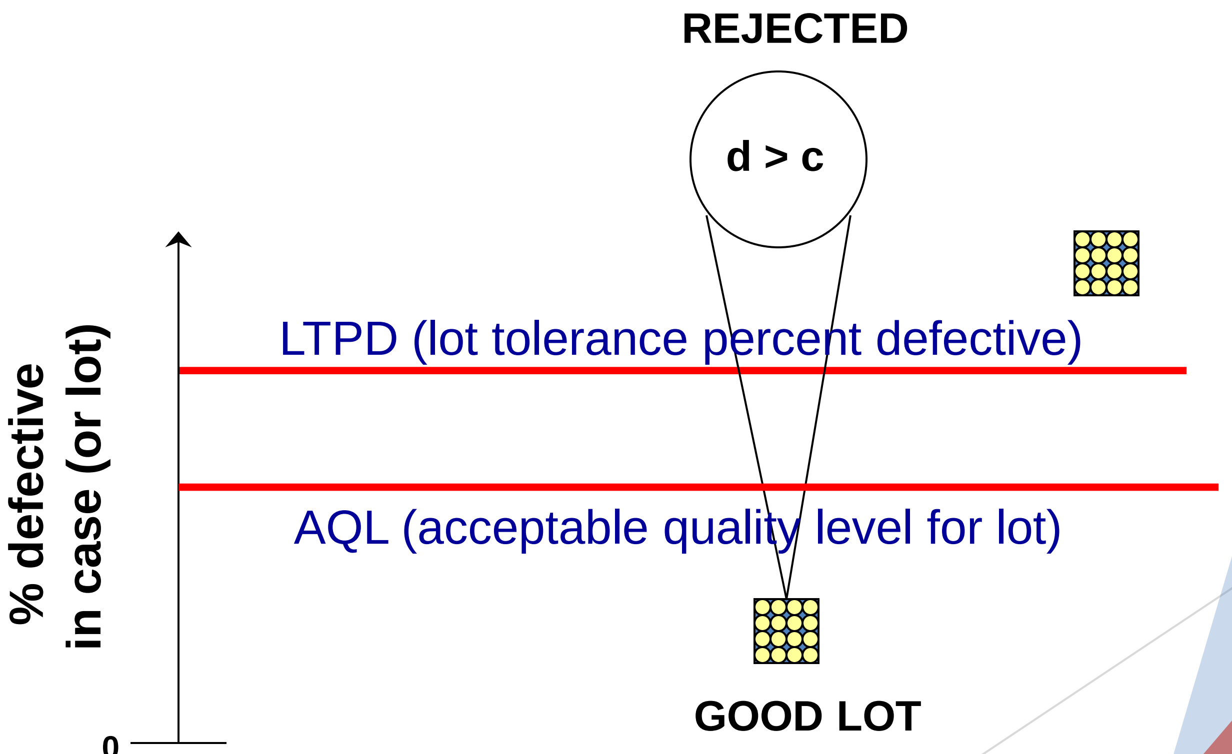
- Each night we inspect cases of cookies produced during the day
- Cases contain 10,000 cookies each
- Cookies are randomly removed from each case & inspected
- Entire cases of cookies are accepted or rejected based on the quality of the samples taken

Acceptance Sampling

Now let's take a look at the 3 cases of cookies produced today…



Acceptance Sampling



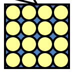
Probability
of committing a Type I error
(producers risk, α)

Acceptance Sampling

ACCEPTED

d ≤ c

BAD LOT



Probability of committing a Type II error (consumers risk, β)

LTPD (lot tolerance percent defective)

AQL (acceptable quality level for lot)

% defective in case (or lot)

0

But… how many cookies should we pull from each case (sample size, n)?

And what is the most defective cookies we can find in a sample before rejecting a lot (c) ?

Wrapup

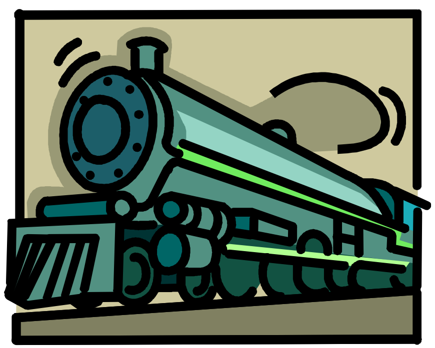
- We looked at SPC and how to use control charts to monitor processes

…basically an in-control process was one with random process variation that varied within some control limits

Variability

As variability is reduced…

quality improves



Always on time…

schedules can be planned more precisely

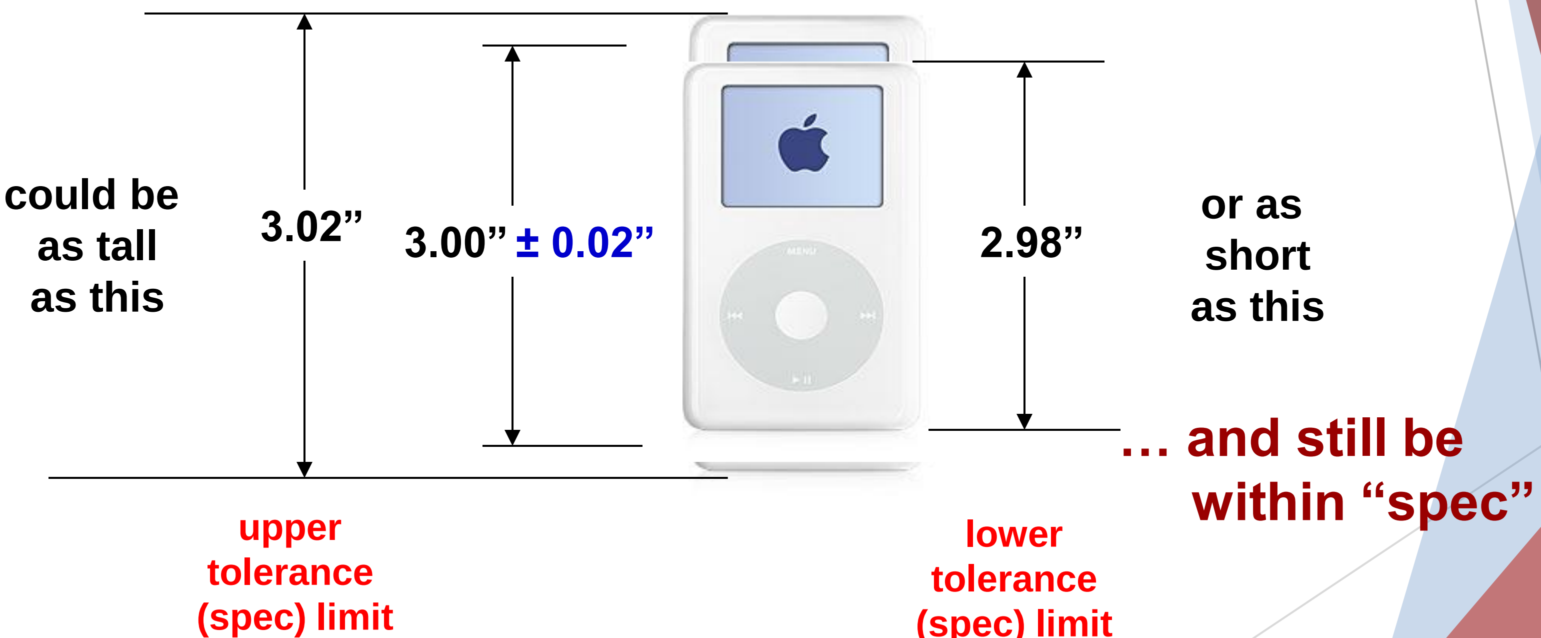
Always consistent sizes…

time can be saved by ordering from catalogs

Unfortunately its impossible to obtain zero variability…

Variability

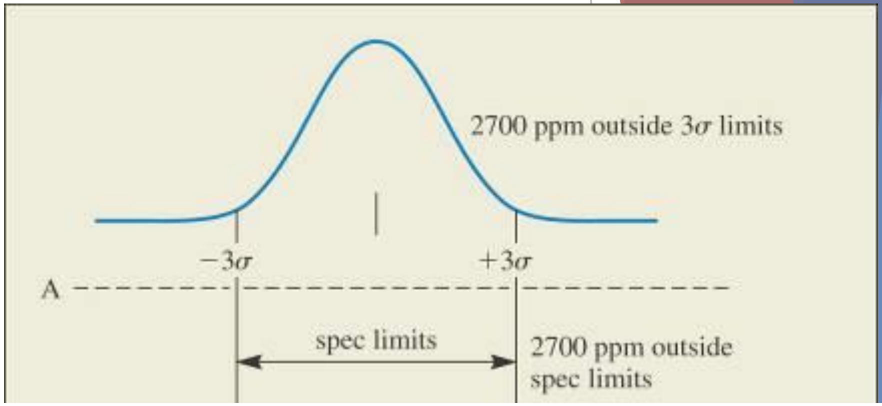
- Designers recognize this
- Provide acceptable limits around target dimensions



BUT, is a part that is 3.0199” that much different than one that’s 3.0201”?

Six Sigma

- A philosophy and set of methods used to reduce variation in the processes that lead to product defects



- The name, “six sigma” refers to the designing spec limits six standard deviations from the process mean

Summary

- Basic concept of Statistical Quality Control
- Sources of statistical variations
- Types of measurements
  - Attributes
  - Variables
- Types of Control Charts
  - Where would each be best used
- What is acceptance sampling
  - Why important
- Six Sigma's basic goal