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FINDING MINIMAL COST PROOFS FOR COST-BASED ABDUCTION

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Abstract

Abduction is the process of proceeding from data describing a set of observations or events, to a set of hypotheses which best explains or accounts for the data. Cost-based abduction (CBA), also called cost-based hypothetically reasoning, is a formalism in which evidence to be explained is treated as a goal to be proven, proofs have costs based on how much needs to be assumed to complete the proof, and the set of assumptions needed to complete a least-cost proof is taken as the best explanation for the given evidence. This project explores a new heuristic approach to finding least-cost proofs for CBA systems and compares the performance against previous published results.

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I. Introduction

Abduction is the process of proceeding from data describing observations or events, to a set of hypotheses, which best explains or accounts for the data [1]. Cost-based abduction (CBA) is an important problem in reasoning under uncertainty [2]. Finding Least-Cost Proofs (LCP's) for CBA systems is known to be NP-hard [3] and has been a subject of considerable research over the past decade [4]. A CBA system is a knowledge representation in which a given world situation is modeled as a 4-tuple where $K = (H, R, c, G)$ where

- H is a set of hypotheses or propositions,
- R is a set of conjunctive rules of the form

  $\left(h_{i_1} \wedge h_{i_2} \wedge \cdots \wedge h_{i_n}\right) \rightarrow h_{i_q}$,

  where $h_{i_1} \wedge h_{i_2} \wedge \cdots \wedge h_{i_n}$ (called the antecedents, with *n* the number of antecedents) and h_{i_q} (called the consequent) are all members of H,
- c is a function $c: H \rightarrow \Re^+$, where $c(h)$ is called the assumability cost of hypothesis $h \in H$ and $\Re^+$denotes the positive real numbers,
- $G \subseteq H$ is called the goal set or the evidence.

The objective of CBA is to find the least cost proof (LCP) for the evidence, where the cost of a proof is taken to be the sum of the costs of all hypotheses that must be assumed in order to complete the proof. Any given hypothesis can be made true in two ways: it can be assumed to be true, at a cost of its assumability cost, or it can be proved. If a hypothesis occurs as the consequent of a rule, then it can be proved, at no cost, to be true by making all the antecedents true, either by assumption or by proof.

If a hypothesis does not appear as the consequent of any rule, then it cannot be proved, it can be made true only by being assumed. The cost of a hypothesis can be infinite (∞), which means that it cannot be assumed, it can only be proved through a combination of other hypotheses or rules. One can assume, without loss of generality, that any hypothesis that appears as the consequent of any rule may have infinite assumability cost. Suppose x_q has a finite assumability cost of a, and appears as the consequent of at least one rule. One can add a hypothesis x'_q with assumability cost a, set the assumability cost of x_q to ∞, and add the rule $x'_q \rightarrow x_q$.

Therefore, we consider the hypothesis set H to be partitioned into two subsets: a set of assumable hypotheses H^A, which have finite assumability costs and do not appear as consequents of any rules, and a set of provable hypotheses H^P, which have infinite assumability costs and, hence, can be made true only by being proved.

The Purpose of This Capstone Project

Recent techniques for finding least-cost proofs to CBA systems use a heuristic repair technique to modify an unfeasible solution so that it proves the goal hypothesis. For instance, Chivers et al. [5] have employed a repair technique applied to unfeasible solutions followed by a 1-OPT [5] optimization to aid in finding approximate solutions with biologically inspired approaches. Similarly, Guinn et al. [6] applied an identical repair and 1-OPT optimization technique using membrane computers to find an optimal solution to a test problem. Although the 1-OPT optimization process improves solution quality of individual member significantly [5] within a genetic algorithm, it may also inhibit the diversity of the population overall by favoring immediate lower cost solutions. Therefore, the ***first goal*** of this project was to explore changes to the repair process and

optimization in an effort to promote population diversity and decrease solution cost. It is important that this repair process also work well within small population sizes (20, 50 members) to decrease computation overhead.

Many recent CBA approaches [5,6,7] have relied solely on the problem instance RAA180 (available at www.cbalib.org and reproduced in the Appendix here) for testing. While RAA180 is a useful benchmark CBA instance, it is difficult to make statements about the generality of the new approaches without exploring their effectiveness on other instances. Thus, the ***second goal*** of this project was to apply a revised evolutionary algorithm (EA) to other benchmark CBA instances and compare performance. In order to isolate the overall effects of the evolutionary algorithm, a simulated annealing (SA) algorithm, utilizing an identical repair technique was implemented and contrasted with recent biologically inspired approaches of hierarchical particle swarm optimization (HPSO) [7], evolutionary algorithms [5] and, membrane computers [6].

II. Background

Finding an LCP for an instance of CBA was shown to be NP-hard [2], and even approximating an LCP within a fixed ratio bound of the optimum has been shown to be NP-hard [4]. Being labeled NP-hard indicates that a particular problem belongs to a class of problems (NP) for which there is *no polynomial* algorithm known to be capable of finding a solution and the problem is at least as hard to solve as the other problems in NP [3].

A number of approaches to CBA have been explored including a best-first heuristic search approach proposed by Charniak and Shimony [8], an admissible heuristic for finding minimum-cost proofs presented by Charniak and Husain [9]. Santos [10] explored a method for transforming a CBA instance into a set of linear constraints, which could then be solved by 0-1 integer linear programming (ILP). Santos' ILP approach was followed by several others: Ishizuka and Matsuo [11] presented a method called slide down and shift up, which uses a combination of linear programming and nonlinear programming to find near-optimal solutions in polynomial-time. Ohsawa and Ishizuka [12] presented a method called bubble propagation, which also finds near-optimal solutions in polynomial-time; Matsuo and Ishizuka [13] investigated linear and nonlinear programming approaches to CBA and to more general logical reasoning problems such as satisfiability. Santos and Santos [14] presented sufficient conditions for a CBA instance to be polynomially-solvable based on the idea of totally unimodular matrices; their work has been extended by Ohsawa and Yachida [15].

Abdelbar [16] showed that methods for CBA can be used for belief revision on belief networks. Kato et al. [17] investigated a method for finding LCP's based on binary decision diagrams. Den [18] presented a chart-based method for cost-based abduction.

Kato et al. [19] investigated a search control mechanism for the A^* algorithm for CBA. In 1996, Kato et al. investigated the parallelization of cost-based abduction with parallel best-first search [20]. Also, other approaches to CBA that have been explored include neural networks [21,22], ant colony [21], iterated local search [22], and population-oriented SA approaches [23,24] to cost-based abduction have also been explored. Chivers et al. [5] has explored two biologically inspired techniques for finding approximate solutions. Recently, Guinn et al. [6] has shown membrane computers to be effective at finding optimal solutions for RAA180.

CBA Instance Library and RAA180

A collection of randomly generated CBA instances is available at www.cbalib.org. The most significant instance from this library is RAA180 due to its established difficulty [22] and the amount of existing research using it as a benchmark. The optimum solution for RAA180 was first achieved using Santos' ILP method [10]. Table 1 shows basic characteristics of RAA180 [22], which features 900 rules and 300 hypotheses, 180 of which are finite. The appendix shows a formularization of RAA180's 900 rules and 300 hypotheses. Later it will be explained that RAA180 has a search space containing 2^{180} possible candidate solutions.

Table 1

Basic Characteristics of the CBA Instance RAA180

Number of hypotheses	300
Number of rules	900
Number of assumable rules	180
Rule depth	max: 38, avg: 25.0, median:27
Optimal solution cost	10,821
ILP CPU time (sec)	88,835
ILP tree depth	41
ILP nodes	178,313

Comparison of Recent Approaches

Table 2 shows results reported by Guinn et al. [6] using a biologically inspired approach known as membrane computing, which is capable of finding optimal solutions for RAA180. Membrane computers (MC) have large computational overhead but reliably provide the lowest cost solution to RAA180.

Table 3 illustrates results from Chivers et al. [5], the best results using an evolutionary algorithm (EA) with population sizes of 10, 20, 50, and 100 shown as "EA(10)", "EA(20)", "EA(50)", "EA(100)", respectively. The best EA results are within 98% of optimal for RAA180. Graph 1 provides a comparison of the solution qualities of the EA, HPSO, and MC techniques featured in Table 2 and Table 3. Graph 1 illustrates that MC approach provides the optimal cost solution while EA and HPSO techniques provide similarly higher cost solutions.

Table 2

1-7-3 CBA Membrane Computer Experiments

Number of Iterations	Mean Score	Min Score	% of the Optimum
100	12158	12011	89.00
200	11497	11100	94.12
300	11423	11330	94.73
400	11084	11019	97.62
500	11087	10972	97.60
600	11062	11019	97.82
700	11036	10977	98.05
800	11019	11019	98.20
900	11059	10994	97.85
1000	10929	10821	99.01

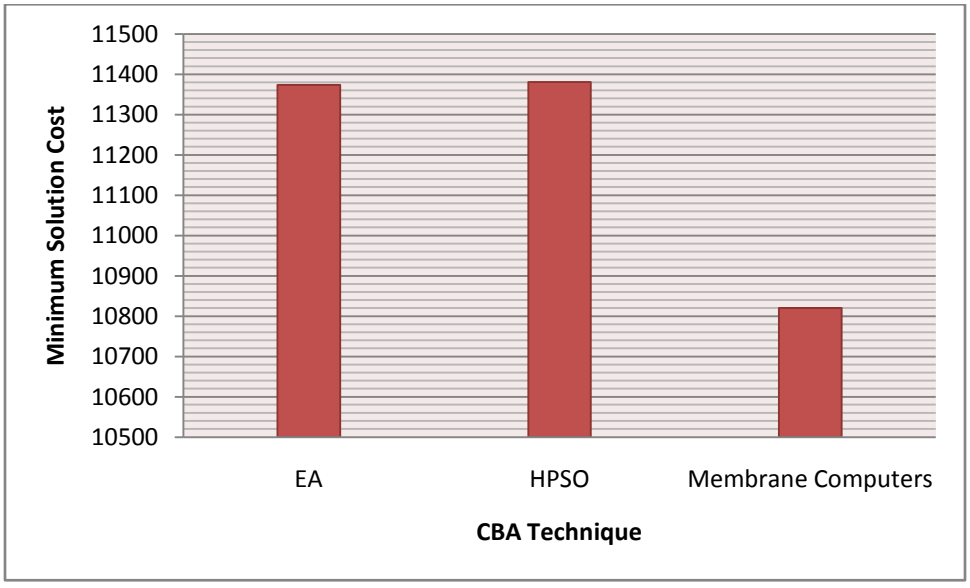
Table 3

Basic Results from EAs and HPSO

	EA(10)	EA(20)	EA(50)	EA(100)	HPSO
No. of Trials	1,365	1,204	490	543	3,584
Mean Cost	11,675.62	11,602.58	11,587.43	11,574.97	12,155.00
Std. Dev Cost	248.09	174.35	149.40	134.78	350.55
Min Cost	11,374	11,374	11,374	11,374	11,381
Median Cost	11,554	11,521	11,521	11,521	12,119
Max Cost	12,960	12,351	12,351	12,070	14,150

Graph 1

Comparison Minimum Solution Cost EA, HPSO and Membrane Computers

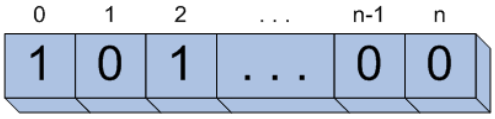


CBA within an Evolutionary Algorithm

A candidate solution can be expressed as a binary string x with n elements, where n is equal to the cardinality of the set of assumable hypotheses H^A. In this arrangement, each element x_i within x corresponds to a finite cost hypothesis and can be assigned either a value of 0 or 1. A value of 0 indicates that a hypothesis x_i is ***excluded*** from the candidate solution whereas a value of 1 indicates that a hypothesis x_i is ***included*** in the candidate solution. It should be noted that the search space for a given problem is 2^n. For the EA implementation discussed in this paper, this binary string representation was used, see Figure 1, and for RAA180 $n = 180$ and the search space contains 2^{180} possible candidate solutions.

Figure 1

A Single Candidate Solution Represented as a Bit String

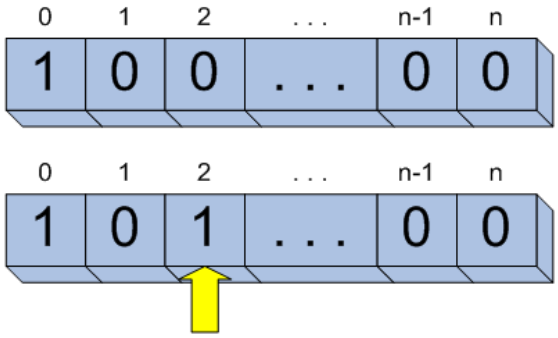


It should be noted that not every possible candidate solution will prove the goal and techniques to generate or alter candidate solutions often lead to infeasible solution. In previous work, Chivers et al. [5] explored both a constant penalty as well as a repair technique based on a type of stochastic local search in the context of a HPSO approach. During the HPSO approach, all solutions were examined to determine if a solution proved the goal. Candidate solutions which did not prove the goal were forced to prove the goal by randomly choosing a 0-bit in a solution and changing it to a 1-bit until the goal was provable. Figure 2 demonstrates a single step of the repair process; in this

example the hypothesis represented by bit location 2 is altered to be ***included*** in the candidate solution.

Figure 2

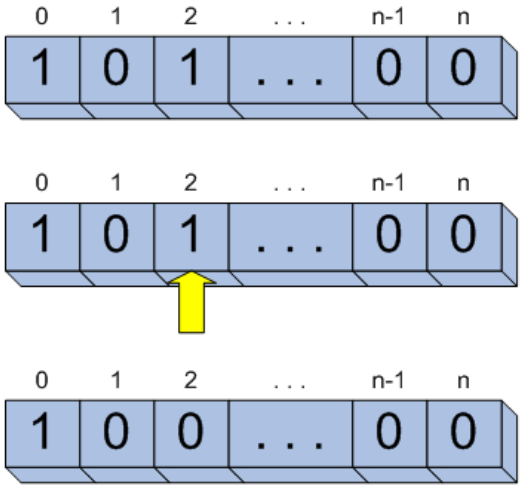
Adding a Single Hypothesis to a Solution



To help compensate for unnecessarily assumed hypotheses a 1-OPT optimization was applied to solutions. Each solution repaired was examined and 1-bits were selected in random order and assigned a 0; if the goal can still be proven, then the bit remained 0, otherwise it was set back to 1. Figure 3 illustrates an example of a bit being altered by the 1-OPT process, ***excluding*** it from the candidate solution.

Figure 3

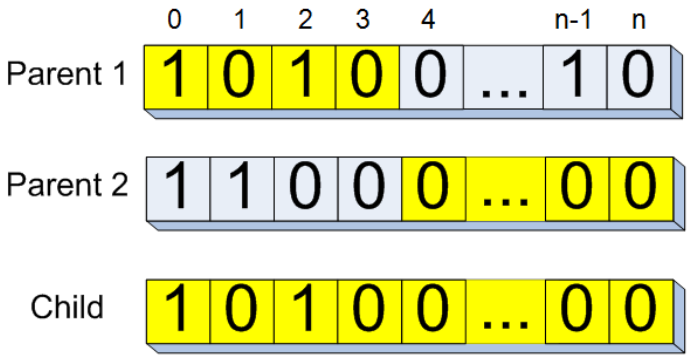
1-OPT Removing a Single Hypothesis



A genetic algorithm (GA) is a type of EA inspired by biological concepts of artificial selection, mutation, chromosomal crossover and recombination. Artificial selection within a GA is modeled by the selection of parent solutions based on a system which gives preference for preferred solutions. Mutation is mimicked by random perturbations to children solutions. Crossover and recombination works analogous to swapping of genes (bits) from parent chromosomes (bit strings). Figure 4 illustrates single point crossover of two parent candidate solutions yielding a single child candidate solution.

Figure 4

Single Point Crossover Yielding a Single Child



III. Methodology

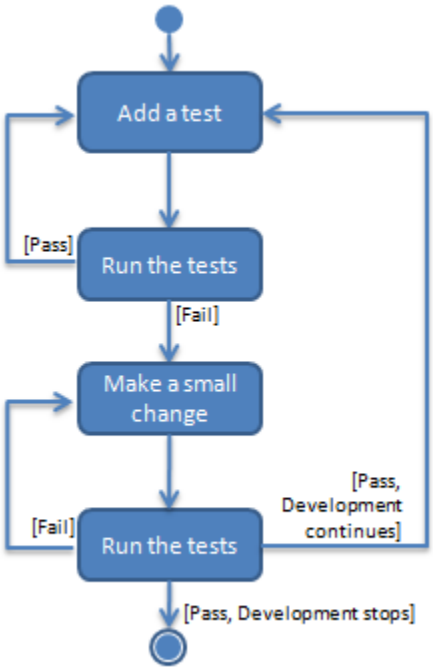
Before describing the experimental design, the software development methodology employed for testing will be illustrated.

Software

All required software was implemented using a Test-Driven Development (TDD) model [25]. This method was elected because the adaptable nature of TDD is conducive to research. TDD involves writing code then refining the code so it passes a test. For the purpose of this project the tests employed emphasized the low cost of candidate solutions, validity of code and characteristics likely to be desirable within an evolutionary algorithm. Figure 5 shows the flow of the TDD process.

Figure 5

Flow of Test Driven Development [25]



Exploring New Heuristic Repair Techniques

As a first approach, I compared candidate solutions using a uniformly distributed random optimization approach proposed by Chivers et al. [5] to a greedy optimization ranking hypotheses by cost. Table 4 illustrates the difference between unrepaired, uniform random and greedy optimization all of the results are significantly far from the optimal cost of 10,821. Although greedy optimization appears to be an improvement over random repair with respect to the quality of solutions, it has a narrow standard deviation that suggests it may be less desirable for promoting diversity within an evolutionary algorithm.

Table 4

Comparison of Repair Techniques

	Unrepaired	Uniform Random	Greedy by Cost
No. Solutions	100	100	100
Min Cost	51,757	19,921	13,132
Mean Cost	70,788.8	40,084.78	18,788.2
Median Cost	70,903	30,406.5	18,611.5
Max Cost	88,566	87,448	24,600
Std. Dev Cost	8,551.3	21,826.2	2,315.0

Periodically applying a not operator after crossover within an EA to increase diversity was also explored. Periodical Not (PN) mutation operator was applied at different occurrence schedules. PN may be considered as a form of mutation operator.

Various schedules applying a not operator were explored, a 25% application of a not operator applied to 10% of randomly selected bits in the solution (selected with uniform probability) after crossover was elected; thus, each bit had a 2.5% probability of being mutated. Table 5 shows the results of periodic application of not operator (PN) within an EA of population size 10.

Another periodic schedule which was attempted to increase diversity was periodic application of the 1-OPT optimization (PO). In the PO schedule, 25% of the cases a 1-OPT optimization is applied. Table 5 shows the results of PO within an EA of population size 10.

Both PN and PO techniques offer improvement over previous EA approaches even within an EA featuring small size population. PO features the greatest standard deviation of cost suggesting it may promote diversity better than PN.

Table 5

Comparison of PN and PO Techniques in an EA of Population 10 for 2000 Generations

	PN	PO
No. Solutions	10	10
Min Cost	11,019	10,940
Mean Cost	11,149.5	11,075.5
Median Cost	11,139.5	11,184.8
Max Cost	11,276	11,751
Std. Dev Cost	93.19	269.85

Applying Evolutionary Algorithm to Cost-based Abduction

The GA selected for this capstone extends previous work done by Chivers et al. [5]. Each generation in the GA consists of several breeding cycles, and at the beginning of each generation the population is sorted by fitness. In order to improve population diversity and discourage premature convergence, any solution that appears more than once in the population is removed during the sorting process and replaced with a solution that is randomly constructed as follows: each assumable hypothesis is assumed with a 0.8 probability; then, a heuristic repair process is applied. After sorting and duplicate solution replacement, the most-fit 20% of the population is identified, and is protected from being replaced by new population elements during the current generation.

In each breeding cycle, two individuals are chosen randomly from anywhere in the population. Crossover is performed using 1-point crossover (crossover point randomly chosen with uniform distribution) to produce two new individuals. For each bit, there is a 2.5% chance a not operator is applied to randomly selected bits in the solution (selected with uniform probability). Each of the two new individuals is then repaired, if necessary. In 50% of the cases 1-OPT optimization is applied; the other 50% solutions are left unaffected. Then, the most-fit (lower-cost) of the two is placed into the population, replacing one of the bottom 80% of the population. Members of the bottom 80% are replaced in order from best to worst with new population elements.

The number of breeding cycles in each generation is equal to 80% of the size of the population, so that when the last slot of the bottom 80% is filled; breeding for that generation is complete. The population is only re-sorted by fitness at the beginning of each generation. Note that new offspring are immediately introduced in the population, which means they can be selected as a parent in the very next breeding cycle.

A New Repair Technique Using Simulated Annealing

First introduced by Kirkpatrick et al. [26], the Simulated Annealing (SA) algorithm is inspired by real-world principles of annealing materials and is used to find heuristic solutions to complex optimization problems. With SA algorithms, a solution is perturbed and accepted (or not) based on the change in solution quality. The magnitude of the perturbations is regulated by temperature variable T that is decreased according to a "simulated" annealing schedule. As temperature variable decreases the acceptance of solutions lowers.

Since the introduction of the SA algorithm it has been applied to a variety of optimization problems including CBA. Abdelbar and Amer [23] applied SA to RAA180 which featured a heuristic repair technique which differs from the EA implementation described after the breeding cycle.

In order to better understand the effects the repair within an EA, equal number of SA trials were attempted. The temperature decreasing factor of 0.999 and an initial temperature $T_0 = 720$ were selected to match previous implementation used by Abdelbar [24]. Each solution perturbation is a not operator that is applied to 10% of randomly selected bits in the solution (selected with uniform probability). In keeping with the EA implementation, each individual is then repaired, if needed. In 50% of the cases 1-OPT optimization is applied; the other 50% solutions are left unaffected. The SA implemented was run for 40,000 iterations. This makes the SA comparable to EA of population of 20 with 2000 generations.

IV. Results

Several of the above techniques where combined for the final implementation including Periodic Not (PN) and Periodic 1-OPT Optimization (PO), the abbreviation PNO is used to indicate where both PN and PO are used. The PNO technique was applied within an EA using two different population sizes (20 and 50) on six different CBA instances from the CBA Instance Library (available at www.cbalib.org). The instances elected were RAA180, OAA110, QAB030, LAB070, CAA200, and RAB050. Out of 425 instances randomly generated these instances were selected because they were determined using integer linear programming (ILP) to be the most challenging. [27]. The best metric to measure relative difficulty of the instances is ILP CPU time, which is the amount of time required to solve each instance using the lp-solve engine [22]. Table 7 shows the characteristics of these problems [27].

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Table 6

Characteristics of RAA180, CAA200, OAA110, QAB030, LAB070, and RAB050

Instance Name:	RAA180	CAA200	OAA110	QAB030	LAB070	RAB050
Number of hypotheses:	300	300	300	100	100	100
Number of rules:	900	900	900	300	300	300
Assumable hypotheses:	180	200	110	30	70	50
Rule depth: max:	38	33	47	32	21	25
avg:	25	21	31.1	18.7	12.3	14.4
median:	27	22	34	20	13	15.5
Optimal solution cost:	10,821	7,678	6,856	3,830	5,423	2,644
ILP CPU time:	88,835	7,605	1,792	208	108	4
ILP tree depth:	41	31	35	21	19	10
ILP nodes:	178,313	6,033	6,675	833	727	45

RAA180

RAA180 ranks the highest in difficulty among the instances from the CBA Instance Library. Graph 2 shows how PNO with population 100 compares to previous work attempting to find least-cost proofs using an EA [5] with population 100 and a HPSO [7]. It is clear from the graph that PNO's entire distribution is significantly less than these previously published results.

Graph 2

Comparison Minimum Solution Cost-- PNO EA, EA and HPSO

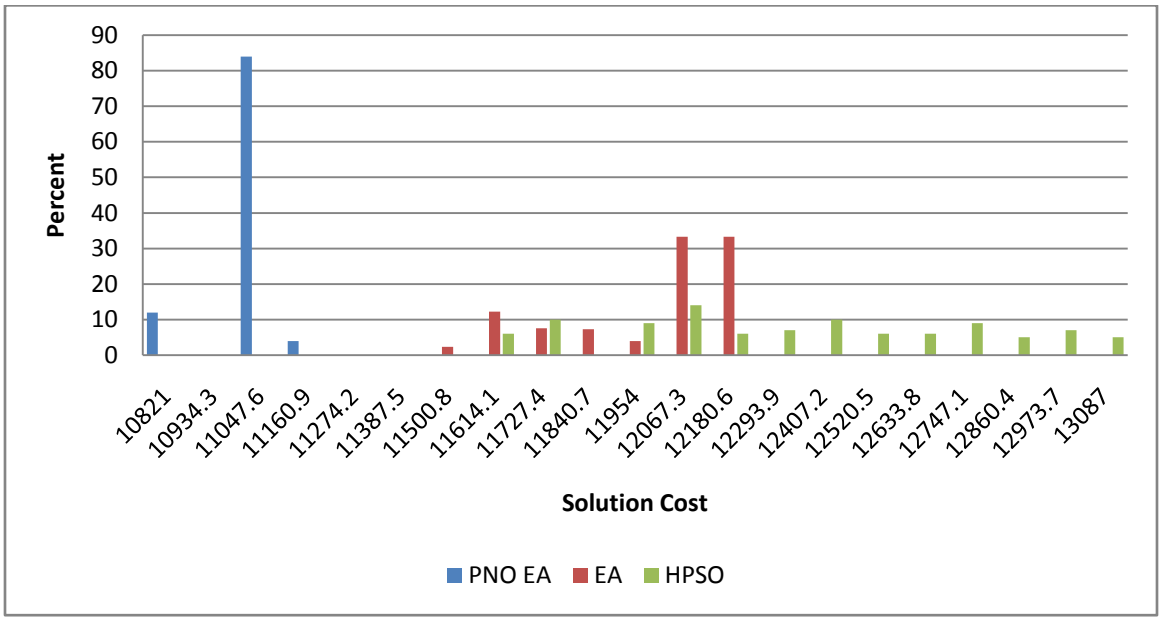


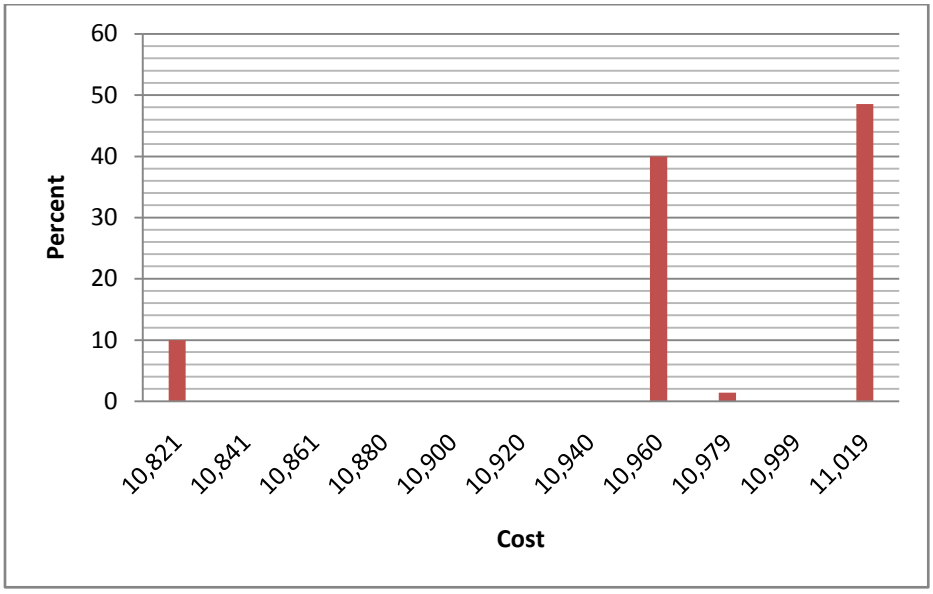
Table 7

Comparison of RAA180 Results

	PNO EA(20)	PNO EA(50)	PNO EA(100)	PNO SA
Instance	RAA180	RAA180	RAA180	RAA180
No. Trials	50	50	100	50
Iterations	2,000	2,000	1,000	40,000
Min Cost	10,821	10,821	10,821	11,269
Mean Cost	10,966.5	10,972.04	10,964.54	11,867
Median Cost	10,954	10,989	10,940	11,807.12
Max Cost	11,019	11,235	11,059	12,234
Std. Dev Cost	61.27	83.18	66.38	237.57
Mean Time (secs)	943	2,031	2,522	1,537
Std Dev Time (secs)	276	714	821	24

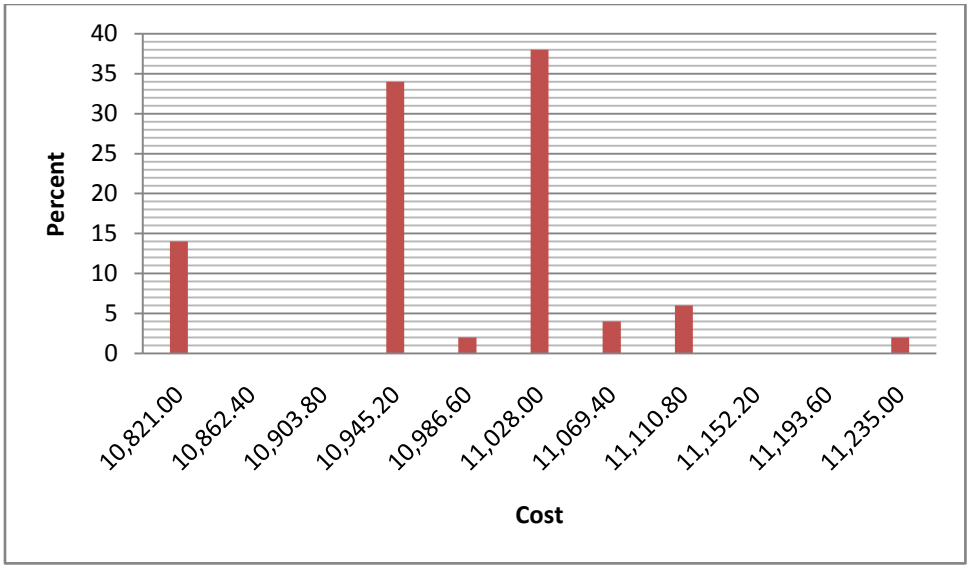
Graph 3

Distribution of PNO EA(20) solutions for RAA180



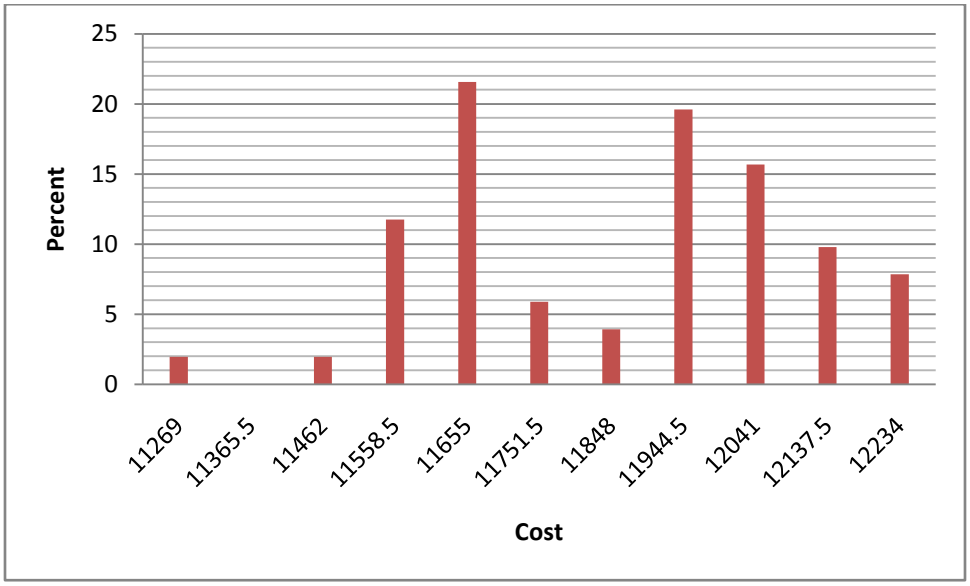
Graph 4

Distribution of PNO EA(50) solutions for RAA180



Graph 5

Distribution of PNO SA solutions for RAA180



CAA200, 0AA110, QAB030, LAB070, and RAB050 Problem - Instances

In addition to RAA180, five other problem instances (CAA200, OAA110, QAB030, LAB070, and RAB050) were tested using both PNO EA and PNO SA approaches. For each problem instance 50 trials were attempted.

The PNO EA implementations with population 20 “PNO EA(20)” and 50 “PNO EA(50)” were capable of finding an optimal cost solution for each of the five attempted problem instances. For problem instances QAB030, LAB070, and RAB050 an optimal cost solution was found consistently regardless of the technique employed.

CAA200 ranks as the second hardest problem in the CBA Instance Library. Table 9 shows PNO EA(20) and PNO EA(50) are both capable of achieve minimum (7,678). This problem is significant because PNO SA was incapable of achieving an optimal cost solution in any of the trials attempted.

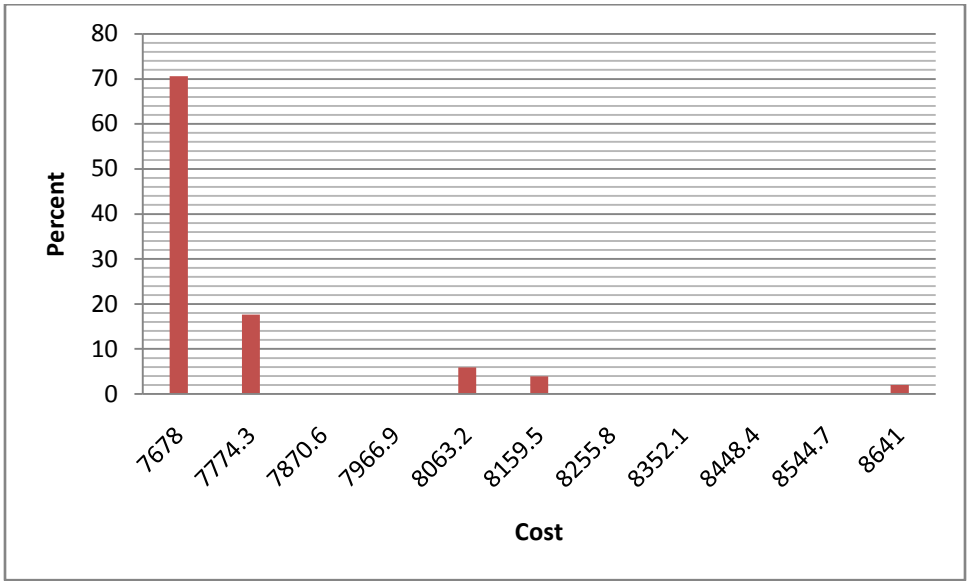
Table 8

CAA200 Results

	PNO EA(20)	PNO EA(50)	PNO SA
Instance	CAA200	CAA200	CAA200
Iterations	2,000	2,000	40,000
No. Trials	50	50	50
Min Cost	7,678	7,678	7,828
Mean Cost	7,678	8,510	8,620
Median Cost	7,736.80	8,485.88	8,554.45
Max Cost	8,641	8,969	9,238
Std. Dev Cost	176.78	308.22	324.70
Mean Time (secs)	441	1,470	1,470
Std Dev Time (secs)	16	2	2

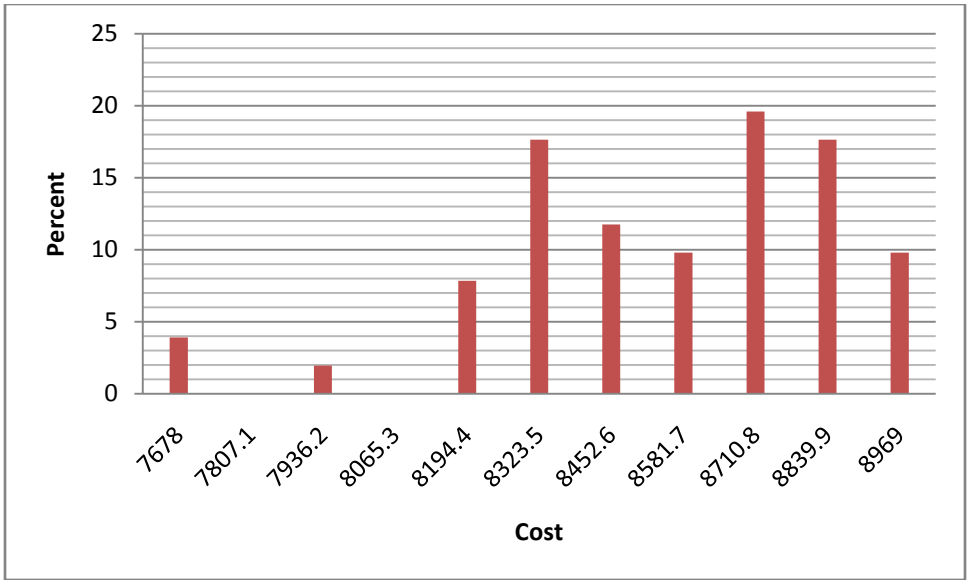
Graph 6

Distribution of PNO EA(20) solutions for CAA200



Graph 7

Distribution of PNO EA(50) solutions for CAA200



Graph 8

Distribution of PNO SA solutions for CAA200

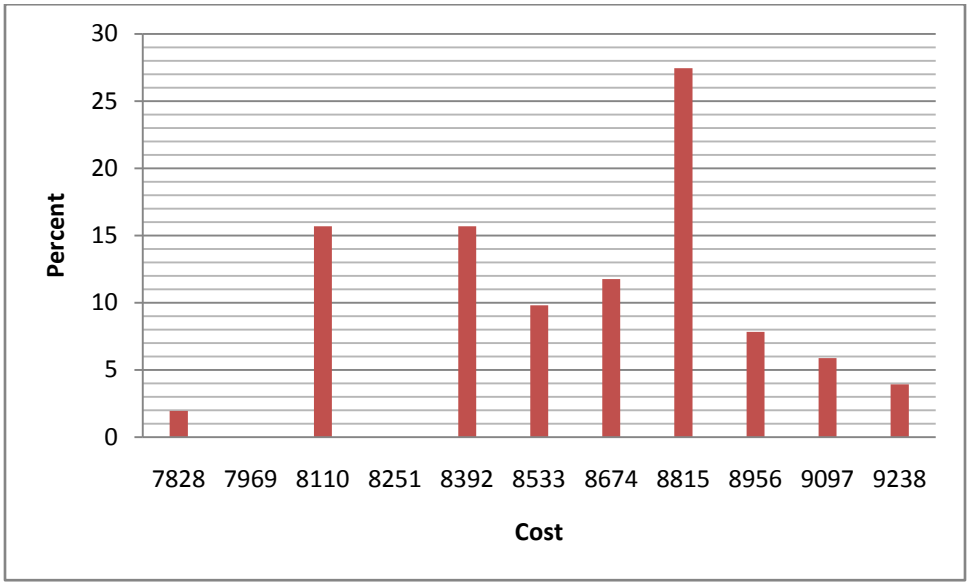


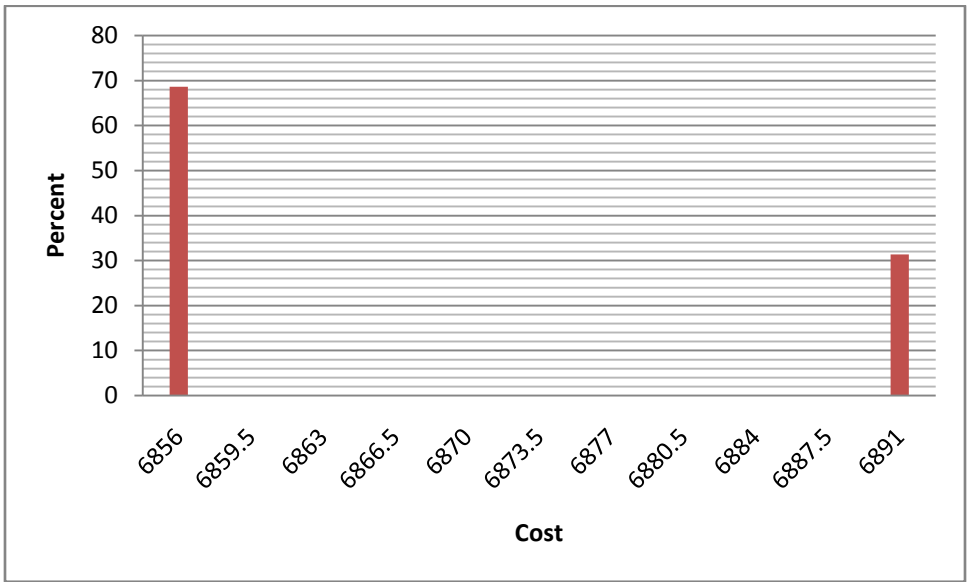
Table 9

OAA110 Results

	PNO EA(20)	PNO EA(50)	PNO SA
Instance	OAA110	OAA110	OAA110
No. Trials	50	50	50
Iterations	2,000	2,000	40,000
Min Cost	6,856	6,856	6,856
Mean Cost	6,856	6,970	6,970
Median Cost	6,866.98	7,019.67	7,016.49
Max Cost	6,891	7,364	7,364
Std. Dev Cost	16.40182	131.26	125.41
Mean Time (secs)	288	1,047	1,048
Std Dev Time (secs)	21	2	2

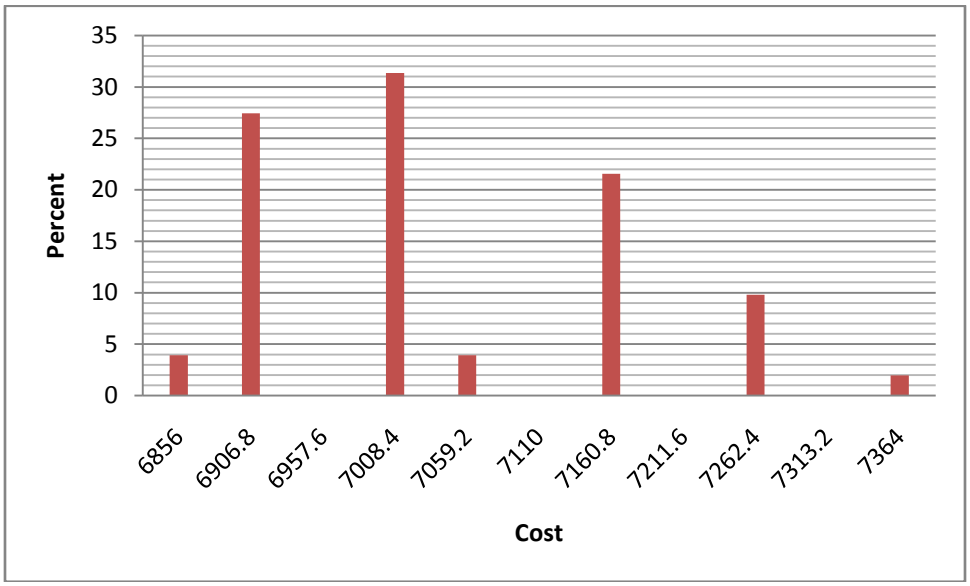
Graph 9

Distribution of PNO EA(20) solutions for QAA110



Graph 10

Distribution of PNO EA(50) solutions for QAA110



Graph 11

Distribution of PNO SA solutions for QAA110

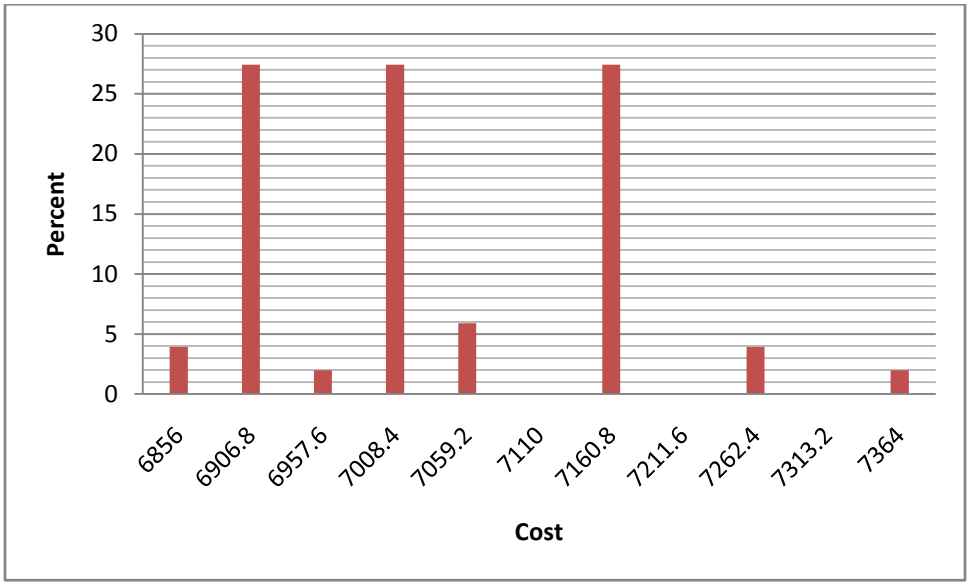


Table 10

QAB030 Results

	PNO EA(20)	PNO EA(50)	PNO SA
Instance	QAB030	QAB030	QAB030
No. Trials	50	50	50
Iterations	2,000	2,000	40,000
Min Cost	3,830	3,830	3,830
Mean Cost	3,830	3,830	3,830
Median Cost	3,830	3,830	3,830
Max Cost	3,830	3,830	3,830
Std. Dev Cost	0	0	0
Mean Time (secs)	22	74	74
Std Dev Time (secs)	1	0	0

Table 11

LAB070 Results

	PNO EA(20)	PNO EA(50)	PNO SA
Instance	LAB070	LAB070	LAB070
No. Trials	50	50	50
Iterations	2,000	2,000	40,000
Min Cost	5,423	5,423	5,423
Mean Cost	5,423	5,423	5,423
Median Cost	5,423	5,423	5,423
Max Cost	5,423	5,423	5,423
Std. Dev Cost	0	0	0
Mean Time (secs)	52	155	155
Std Dev Time (secs)	1	0	1

Table 12

RAB050 Results

	PNO EA(20)	PNO EA(50)	PNO SA
Instance	RAB050	RAB050	RAB050
Iterations	2,000	2,000	40,000
No. Trials	50	50	50
Min Cost	2,644	2,644	2,644
Mean Cost	2,644	2,644	2,644
Median Cost	2,644	2,644	2,644
Max Cost	2,644	2,644	2,644
Std. Dev Cost	0	0	0
Mean Time (secs)	45	109	108
Std Dev Time (secs)	1	1	1

V. Conclusion and Future Work

In this paper, an improved repair and optimization approach called PNO was introduced and applied within an EA with population sizes of 20 and 50, to several unique CBA problem instances (RAA180, OAA110, QAB030, LAB070, CAA200, and RAB050). The PNO technique was then applied within a SA to the same unique problem instances as the EA. The performance of the EA and the SA were compared to reported performance results of the HPSO, EA, SA, and Membrane Computing. It was shown that the newly proposed PNO EA approach was capable of achieving good quality solutions but with added computational cost. Results from trials conducted on several CBA problem instances suggest that this technique is applicable to the general family of problems.

There are opportunities for future work improving the computational time, possibly by tuning the schedules for PO and PN. Other future work could establish more difficult problem instances beyond what are available from the CBA Instance Library.

VI. Timeline

Spring 2010

- Gained understanding of capstone requirements, form capstone committee and complete required forms.
- Complete proposal for capstone.
- Deliver capstone proposal presentation.

Summer 2010

- Explored the relative merits of existing heuristic repair techniques.
- Developed new heuristic repair techniques.
- Developed software to conduct experiments of both evolutionary algorithm approach and simulated annealing approach featuring new heuristic repair techniques.

Fall 2010

- Conducted experiments using software developed in the summer.
- Drew conclusions about experimental results and contrasted these with existing results.
- Completed capstone documentation.
- Presented concluding capstone presentation.

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VIII. Appendix

RAA180 Hypothesis Cost Table

$c(1)=273$
$c(2)=\infty$
$c(3)=746$
$c(4)=\infty$
$c(5)=799$
$c(6)=\infty$
$c(7)=195$
$c(8)=\infty$
$c(9)=444$
$c(10)=\infty$
$c(11)=855$
$c(12)=432$
$c(13)=150$
$c(14)=\infty$
$c(15)=223$
$c(16)=734$
$c(17)=\infty$
$c(18)=543$
$c(19)=616$
$c(20)=810$
$c(21)=\infty$
$c(22)=134$
$c(23)=942$
$c(24)=567$
$c(25)=\infty$
$c(26)=769$
$c(27)=288$
$c(28)=106$
$c(29)=249$
$c(30)=115$
$c(31)=\infty$
$c(32)=199$
$c(33)=\infty$
$c(34)=\infty$
$c(35)=580$
$c(36)=\infty$
$c(37)=794$
$c(38)=837$
$c(39)=\infty$
$c(40)=751$

c(41)=896
c(42)=726
c(43)=∞
c(44)=655
c(45)=∞
c(46)=120
c(47)=393
c(48)=2
c(49)=∞
c(50)=268
c(51)=150
c(52)=799
c(53)=381
c(54)=∞
c(55)=∞
c(56)=95
c(57)=557
c(58)=349
c(59)=∞
c(60)=559
c(61)=88
c(62)=∞
c(63)=∞
c(64)=561
c(65)=∞
c(66)=∞
c(67)=∞
c(68)=31
c(69)=∞
c(70)=358
c(71)=664
c(72)=653
c(73)=∞
c(74)=234
c(75)=516
c(76)=192
c(77)=∞
c(78)=149
c(79)=750
c(80)=990
c(81)=349
c(82)=128
c(83)=137
c(84)=949

c(85)=∞
c(86)=144
c(87)=∞
c(88)=518
c(89)=173
c(90)=∞
c(91)=600
c(92)=∞
c(93)=724
c(94)=921
c(95)=963
c(96)=∞
c(97)=∞
c(98)=∞
c(99)=379
c(100)=60
c(101)=∞
c(102)=∞
c(103)=318
c(104)=504
c(105)=502
c(106)=878
c(107)=130
c(108)=∞
c(109)=252
c(110)=∞
c(111)=∞
c(112)=∞
c(113)=∞
c(114)=682
c(115)=749
c(116)=∞
c(117)=∞
c(118)=∞
c(119)=∞
c(120)=837
c(121)=896
c(122)=∞
c(123)=497
c(124)=80
c(125)=126
c(126)=282
c(127)=467
c(128)=439

c(129)=924
c(130)=600
c(131)=184
c(132)=147
c(133)=∞
c(134)=∞
c(135)=∞
c(136)=842
c(137)=900
c(138)=870
c(139)=419
c(140)=349
c(141)=677
c(142)=757
c(143)=∞
c(144)=∞
c(145)=8
c(146)=∞
c(147)=∞
c(148)=∞
c(149)=316
c(150)=720
c(151)=953
c(152)=629
c(153)=∞
c(154)=187
c(155)=∞
c(156)=∞
c(157)=∞
c(158)=769
c(159)=∞
c(160)=196
c(161)=∞
c(162)=594
c(163)=427
c(164)=7
c(165)=428
c(166)=∞
c(167)=129
c(168)=∞
c(169)=7
c(170)=875
c(171)=∞
c(172)=363

c(173)=∞
c(174)=∞
c(175)=863
c(176)=∞
c(177)=826
c(178)=874
c(179)=351
c(180)=749
c(181)=∞
c(182)=∞
c(183)=337
c(184)=50
c(185)=605
c(186)=∞
c(187)=∞
c(188)=∞
c(189)=298
c(190)=503
c(191)=224
c(192)=∞
c(193)=968
c(194)=∞
c(195)=∞
c(196)=187
c(197)=178
c(198)=501
c(199)=∞
c(200)=∞
c(201)=674
c(202)=10
c(203)=78
c(204)=477
c(205)=725
c(206)=∞
c(207)=963
c(208)=174
c(209)=850
c(210)=396
c(211)=∞
c(212)=∞
c(213)=406
c(214)=∞
c(215)=∞
c(216)=∞

c(217)=∞
c(218)=6
c(219)=∞
c(220)=∞
c(221)=∞
c(222)=804
c(223)=438
c(224)=∞
c(225)=299
c(226)=∞
c(227)=∞
c(228)=57
c(229)=∞
c(230)=∞
c(231)=843
c(232)=∞
c(233)=638
c(234)=∞
c(235)=639
c(236)=707
c(237)=∞
c(238)=559
c(239)=324
c(240)=∞
c(241)=981
c(242)=378
c(243)=626
c(244)=445
c(245)=∞
c(246)=∞
c(247)=∞
c(248)=∞
c(249)=472
c(250)=376
c(251)=∞
c(252)=391
c(253)=∞
c(254)=201
c(255)=543
c(256)=823
c(257)=833
c(258)=927
c(259)=959
c(260)=818

c(261)=356
c(262)=∞
c(263)=561
c(264)=∞
c(265)=530
c(266)=∞
c(267)=∞
c(268)=399
c(269)=∞
c(270)=∞
c(271)=∞
c(272)=903
c(273)=∞
c(274)=147
c(275)=700
c(276)=∞
c(277)=∞
c(278)=769
c(279)=∞
c(280)=312
c(281)=483
c(282)=903
c(283)=924
c(284)=311
c(285)=323
c(286)=420
c(287)=557
c(288)=131
c(289)=804
c(290)=148
c(291)=953
c(292)=∞
c(293)=∞
c(294)=∞
c(295)=960
c(296)=∞
c(297)=∞
c(298)=169
c(299)=∞
c(300)=∞

RAA180 Rules Table (presented in unsorted order)

$42 \wedge 190 \wedge 98 \wedge 63 \wedge 183 \wedge 6 \wedge 102 \rightarrow 248$
$23 \wedge 226 \wedge 24 \wedge 37 \wedge 248 \rightarrow 264$
$11 \wedge 82 \wedge 78 \wedge 17 \wedge 91 \wedge 94 \wedge 28 \rightarrow 97$
$123 \wedge 169 \wedge 57 \wedge 128 \wedge 137 \wedge 41 \rightarrow 176$
$29 \wedge 31 \wedge 9 \wedge 27 \wedge 10 \wedge 24 \rightarrow 34$
$43 \wedge 93 \wedge 12 \wedge 31 \wedge 68 \wedge 90 \wedge 22 \rightarrow 101$
$29 \wedge 22 \wedge 10 \wedge 30 \wedge 12 \wedge 40 \rightarrow 43$
$94 \wedge 100 \rightarrow 224$
$117 \wedge 69 \wedge 171 \wedge 118 \wedge 22 \rightarrow 188$
$77 \wedge 213 \wedge 56 \wedge 131 \wedge 146 \wedge 85 \rightarrow 229$
$213 \wedge 171 \wedge 5 \wedge 181 \wedge 40 \wedge 177 \rightarrow 214$
$67 \wedge 7 \wedge 27 \wedge 35 \wedge 39 \wedge 64 \wedge 17 \wedge 45 \rightarrow 73$
$96 \wedge 192 \wedge 41 \wedge 136 \wedge 30 \wedge 145 \wedge 165 \wedge 273 \rightarrow 292$
$56 \wedge 26 \wedge 1 \wedge 54 \wedge 2 \wedge 29 \wedge 16 \wedge 32 \rightarrow 59$
$69 \wedge 22 \wedge 101 \wedge 6 \wedge 106 \wedge 74 \wedge 90 \wedge 73 \rightarrow 111$
$192 \wedge 188 \wedge 37 \wedge 11 \wedge 165 \wedge 133 \rightarrow 200$
$23 \wedge 69 \wedge 38 \wedge 25 \wedge 50 \wedge 30 \wedge 12 \wedge 22 \rightarrow 90$
$101 \wedge 44 \wedge 121 \wedge 38 \rightarrow 292$
$88 \wedge 28 \wedge 154 \wedge 113 \wedge 62 \wedge 10 \wedge 68 \rightarrow 186$
$56 \wedge 16 \wedge 34 \wedge 22 \wedge 7 \wedge 73 \wedge 67 \wedge 23 \rightarrow 87$
$104 \wedge 94 \wedge 15 \wedge 89 \wedge 39 \wedge 16 \wedge 110 \wedge 59 \rightarrow 112$
$22 \wedge 87 \wedge 236 \wedge 154 \wedge 280 \wedge 202 \wedge 151 \wedge 68 \rightarrow 292$
$27 \wedge 9 \wedge 18 \rightarrow 45$
$180 \wedge 111 \wedge 148 \wedge 63 \wedge 181 \wedge 96 \wedge 55 \rightarrow 270$
$35 \wedge 3 \wedge 41 \wedge 51 \wedge 64 \rightarrow 85$
$119 \wedge 147 \wedge 137 \wedge 255 \rightarrow 271$
$80 \wedge 33 \wedge 25 \wedge 50 \wedge 27 \wedge 12 \wedge 9 \wedge 62 \rightarrow 87$
$26 \wedge 125 \wedge 142 \wedge 213 \wedge 42 \wedge 262 \wedge 1 \wedge 223 \rightarrow 270$
$4 \wedge 6 \rightarrow 10$
$62 \wedge 44 \wedge 22 \wedge 10 \rightarrow 63$
$86 \wedge 61 \wedge 18 \rightarrow 174$
$54 \wedge 150 \wedge 190 \wedge 36 \wedge 12 \rightarrow 219$
$179 \rightarrow 186$
$56 \wedge 82 \wedge 114 \wedge 17 \wedge 83 \rightarrow 122$
$50 \rightarrow 97$
$2 \wedge 3 \wedge 5 \wedge 4 \wedge 1 \rightarrow 6$
$108 \wedge 125 \wedge 145 \wedge 8 \wedge 42 \rightarrow 168$
$19 \wedge 87 \wedge 13 \wedge 24 \wedge 18 \wedge 56 \wedge 71 \rightarrow 146$
$67 \wedge 191 \wedge 97 \wedge 146 \wedge 10 \wedge 84 \wedge 101 \wedge 159 \rightarrow 232$
$252 \wedge 84 \wedge 40 \wedge 22 \wedge 181 \wedge 158 \rightarrow 292$
$205 \rightarrow 234$
$106 \wedge 36 \wedge 3 \wedge 32 \rightarrow 117$

48 ⟶ 54
23 ⋀ 39 ⋀ 86 ⋀ 57 ⋀ 136 ⋀ 103 ⋀ 120 ⋀ 135 ⟶146
16 ⋀ 6 ⋀ 1 ⋀ 7 ⋀ 3 ⋀ 8 ⋀ 15 ⟶ 25
10 ⟶ 31
60 ⋀ 19 ⋀ 93 ⋀ 91 ⟶ 113
143 ⋀ 22 ⋀ 191 ⋀ 71 ⋀ 43 ⋀ 106 ⟶ 267
66 ⋀ 11 ⋀ 45 ⋀ 91 ⟶ 92
45 ⋀ 171 ⟶ 211
87 ⋀ 136 ⋀ 187 ⋀ 89 ⋀ 186 ⋀ 160 ⋀ 170 ⋀ 74 ⟶212
220 ⋀ 107 ⋀ 209 ⋀ 10 ⟶ 293
197 ⋀ 143 ⋀ 218 ⋀ 200 ⋀ 98 ⋀ 73 ⋀ 55 ⋀ 16 ⟶219
2 ⋀ 54 ⋀ 51 ⋀ 65 ⋀ 64 ⋀ 43 ⋀ 53 ⟶ 67
67 ⋀ 39 ⋀ 88 ⟶ 97
104 ⟶ 146
118 ⋀ 160 ⟶ 182
33 ⋀ 21 ⟶ 101
29 ⋀ 25 ⋀ 19 ⋀ 15 ⋀ 46 ⋀ 40 ⟶ 59
51 ⋀ 12 ⋀ 1 ⋀ 115 ⋀ 42 ⋀ 101 ⋀ 64 ⟶ 116
135 ⋀ 271 ⋀ 75 ⟶ 297
109 ⋀ 69 ⋀ 62 ⋀ 47 ⋀ 50 ⟶ 117
11 ⋀ 47 ⟶ 195
5 ⟶ 17
48 ⋀ 69 ⟶ 146
112 ⋀ 74 ⋀ 36 ⋀ 25 ⋀ 7 ⋀ 46 ⋀ 56 ⋀ 154 ⟶173
88 ⋀ 10 ⋀ 91 ⋀ 66 ⋀ 86 ⋀ 28 ⋀ 74 ⟶ 102
33 ⋀ 67 ⋀ 147 ⋀ 134 ⟶ 194
47 ⋀ 29 ⋀ 65 ⋀ 38 ⋀ 60 ⋀ 14 ⟶ 66
94 ⟶ 96
44 ⋀ 80 ⋀ 172 ⟶ 181
16 ⋀ 9 ⋀ 2 ⋀ 13 ⋀ 8 ⋀ 1 ⋀ 5 ⟶ 21
83 ⋀ 173 ⋀ 41 ⟶ 215
5 ⋀ 7 ⋀ 1 ⋀ 9 ⟶ 10
128 ⋀ 54 ⋀ 19 ⋀ 140 ⋀ 66 ⋀ 141 ⋀ 155 ⋀ 58 ⟶296
41 ⋀ 7 ⟶ 224
198 ⋀ 176 ⋀ 211 ⋀ 221 ⋀ 79 ⋀ 90 ⋀ 28 ⋀ 152 ⟶224
15 ⋀ 95 ⋀ 89 ⟶ 147
118 ⟶ 174
51 ⟶ 294
76 ⋀ 51 ⋀ 117 ⟶ 192
1 ⟶ 2
114 ⟶ 194
53 ⋀ 73 ⋀ 95 ⋀ 8 ⋀ 58 ⟶ 117
54 ⋀ 162 ⋀ 140 ⋀ 112 ⟶ 171
67 ⋀ 62 ⋀ 165 ⋀ 162 ⋀ 55 ⋀ 85 ⟶ 221

13 ∧ 4 ∧ 3 ∧ 32 ∧ 8 ∧ 16 ⟶ 33
71 ∧ 100 ∧ 112 ∧ 9 ∧ 85 ∧ 128 ∧ 127 ∧ 18 ⟶143
169 ∧ 166 ∧ 90 ∧ 50 ∧ 223 ∧ 48 ∧ 5 ∧ 132 ⟶234
81 ∧ 24 ∧ 46 ∧ 26 ∧ 83 ∧ 75 ∧ 35 ∧ 66 ⟶98
14 ∧ 170 ⟶ 171
66 ∧ 73 ∧ 15 ∧ 62 ∧ 53 ⟶ 77
82 ∧ 33 ∧ 114 ∧ 40 ∧ 44 ∧ 41 ∧ 103 ∧ 126 ⟶143
121 ∧ 69 ∧ 35 ∧ 167 ∧ 195 ∧ 60 ⟶ 224
141 ⟶ 296
63 ∧ 109 ∧ 51 ∧ 159 ∧ 52 ∧ 114 ∧ 156 ∧ 211 ⟶264
59 ∧ 171 ∧ 149 ⟶ 220
47 ∧ 27 ∧ 1 ∧ 7 ∧ 10 ⟶ 54
7 ∧ 10 ∧ 58 ∧ 110 ∧ 143 ⟶ 157
56 ∧ 186 ∧ 123 ∧ 31 ∧ 50 ⟶ 247
35 ∧ 40 ⟶ 96
44 ∧ 10 ∧ 64 ∧ 26 ∧ 8 ∧ 9 ∧ 12 ⟶ 65
40 ∧ 12 ∧ 49 ∧ 46 ∧ 18 ∧ 32 ⟶ 174
164 ∧ 146 ∧ 151 ∧ 76 ⟶ 199
39 ∧ 157 ∧ 25 ∧ 83 ∧ 142 ∧ 96 ∧ 70 ∧ 54 ⟶159
146 ∧ 69 ⟶ 234
107 ∧ 13 ∧ 16 ∧ 4 ∧ 49 ∧ 102 ∧ 44 ⟶ 111
181 ⟶ 230
9 ∧ 11 ∧ 54 ∧ 3 ∧ 35 ⟶ 69
118 ⟶ 195
85 ∧ 33 ∧ 82 ∧ 51 ∧ 95 ∧ 10 ∧ 76 ⟶ 111
118 ∧ 32 ⟶ 229
44 ∧ 57 ⟶ 73
37 ∧ 36 ∧ 78 ⟶ 101
25 ⟶ 45
191 ∧ 69 ∧ 100 ∧ 124 ∧ 37 ⟶ 227
41 ∧ 36 ⟶ 54
91 ∧ 36 ⟶ 147
188 ∧ 33 ∧ 21 ∧ 181 ⟶ 248
135 ∧ 125 ∧ 116 ∧ 83 ∧ 109 ∧ 9 ⟶ 159
132 ∧ 185 ∧ 106 ∧ 51 ⟶ 199
154 ∧ 201 ∧ 88 ∧ 205 ∧ 208 ⟶ 246
152 ⟶ 194
4 ∧ 74 ∧ 107 ∧ 93 ∧ 97 ⟶ 161
123 ∧ 45 ∧ 42 ∧ 27 ⟶ 277
17 ∧ 58 ∧ 50 ∧ 57 ∧ 12 ⟶ 62
16 ∧ 41 ∧ 12 ∧ 107 ⟶ 119
36 ∧ 64 ∧ 53 ∧ 66 ∧ 96 ∧ 103 ⟶ 143
70 ∧ 16 ∧ 42 ∧ 168 ∧ 252 ∧ 101 ⟶ 292
71 ⟶ 292

$18 \wedge 4 \wedge 9 \wedge 11 \wedge 26 \wedge 16 \longrightarrow 33$
$171 \wedge 133 \wedge 98 \wedge 107 \longrightarrow 221$
$48 \longrightarrow 59$
$122 \wedge 15 \wedge 106 \wedge 210 \wedge 140 \wedge 180 \wedge 170 \longrightarrow 214$
$127 \wedge 144 \wedge 81 \wedge 115 \wedge 36 \longrightarrow 186$
$100 \wedge 145 \wedge 89 \wedge 94 \wedge 139 \wedge 58 \wedge 73 \longrightarrow 229$
$140 \wedge 43 \wedge 25 \wedge 53 \wedge 23 \longrightarrow 173$
$187 \wedge 190 \longrightarrow 195$
$57 \wedge 198 \wedge 122 \longrightarrow 214$
$125 \wedge 49 \wedge 133 \wedge 127 \wedge 139 \longrightarrow 186$
$58 \wedge 78 \wedge 7 \wedge 72 \wedge 40 \longrightarrow 90$
$156 \wedge 215 \wedge 78 \wedge 114 \longrightarrow 248$
$56 \wedge 3 \wedge 27 \wedge 57 \longrightarrow 92$
$129 \wedge 132 \wedge 98 \longrightarrow 270$
$76 \wedge 91 \wedge 40 \wedge 105 \wedge 57 \wedge 81 \wedge 97 \wedge 41 \longrightarrow 111$
$1 \longrightarrow 2$
$36 \longrightarrow 49$
$2 \wedge 12 \wedge 5 \wedge 3 \wedge 9 \longrightarrow 21$
$65 \wedge 17 \wedge 164 \longrightarrow 234$
$14 \wedge 19 \wedge 10 \wedge 25 \longrightarrow 31$
$46 \wedge 110 \wedge 82 \wedge 139 \wedge 93 \wedge 54 \wedge 204 \wedge 194 \longrightarrow 224$
$17 \wedge 148 \longrightarrow 199$
$45 \wedge 82 \wedge 80 \longrightarrow 97$
$29 \wedge 121 \longrightarrow 171$
$113 \wedge 103 \longrightarrow 143$
$80 \wedge 16 \longrightarrow 111$
$99 \longrightarrow 113$
$263 \wedge 79 \wedge 26 \wedge 116 \wedge 253 \wedge 160 \wedge 246 \longrightarrow 266$
$156 \wedge 79 \wedge 26 \wedge 149 \longrightarrow 212$
$90 \wedge 12 \wedge 89 \wedge 24 \wedge 61 \wedge 99 \wedge 39 \longrightarrow 135$
$48 \longrightarrow 134$
$3 \wedge 5 \wedge 2 \wedge 10 \wedge 9 \wedge 8 \wedge 1 \wedge 11 \longrightarrow 14$
$38 \longrightarrow 39$
$126 \wedge 105 \wedge 77 \longrightarrow 155$
$9 \wedge 4 \wedge 27 \longrightarrow 31$
$150 \longrightarrow 187$
$59 \longrightarrow 156$
$55 \longrightarrow 59$
$116 \wedge 16 \wedge 56 \longrightarrow 134$
$25 \wedge 68 \wedge 36 \wedge 89 \wedge 48 \wedge 96 \wedge 4 \wedge 40 \longrightarrow 111$
$137 \wedge 55 \longrightarrow 279$
$26 \wedge 170 \wedge 136 \wedge 63 \wedge 95 \wedge 37 \longrightarrow 200$
$31 \wedge 215 \wedge 149 \wedge 3 \wedge 73 \wedge 163 \wedge 180 \wedge 102 \longrightarrow 267$
$159 \wedge 146 \wedge 141 \longrightarrow 232$

$1 \longrightarrow 2$
$132 \wedge 27 \longrightarrow 181$
$108 \wedge 121 \wedge 42 \wedge 186 \wedge 64 \longrightarrow 200$
$138 \wedge 87 \wedge 51 \wedge 80 \wedge 74 \wedge 143 \longrightarrow 147$
$84 \wedge 63 \longrightarrow 90$
$216 \wedge 39 \wedge 75 \wedge 44 \longrightarrow 240$
$20 \wedge 30 \wedge 16 \wedge 14 \wedge 29 \wedge 7 \wedge 12 \longrightarrow 39$
$86 \longrightarrow 111$
$80 \wedge 81 \wedge 38 \wedge 131 \wedge 10 \wedge 13 \wedge 39 \wedge 64 \longrightarrow 144$
$167 \wedge 106 \wedge 157 \wedge 111 \wedge 25 \longrightarrow 174$
$160 \wedge 180 \wedge 210 \wedge 177 \wedge 111 \wedge 108 \wedge 179 \longrightarrow 217$
$11 \wedge 127 \wedge 82 \wedge 6 \wedge 1 \longrightarrow 195$
$29 \wedge 38 \wedge 62 \wedge 28 \wedge 56 \wedge 8 \wedge 33 \wedge 51 \longrightarrow 69$
$4 \wedge 133 \wedge 59 \wedge 146 \wedge 30 \longrightarrow 192$
$22 \wedge 7 \wedge 10 \wedge 21 \wedge 3 \wedge 1 \wedge 4 \wedge 23 \longrightarrow 25$
$246 \wedge 92 \wedge 200 \wedge 64 \wedge 122 \wedge 23 \wedge 141 \wedge 93 \longrightarrow 276$
$7 \wedge 5 \wedge 1 \wedge 3 \wedge 6 \longrightarrow 8$
$80 \wedge 118 \wedge 119 \wedge 62 \longrightarrow 146$
$159 \wedge 73 \wedge 163 \wedge 221 \longrightarrow 227$
$88 \wedge 72 \wedge 38 \wedge 7 \wedge 13 \wedge 62 \wedge 4 \wedge 49 \longrightarrow 122$
$40 \longrightarrow 69$
$89 \wedge 11 \longrightarrow 267$
$12 \wedge 4 \wedge 15 \longrightarrow 33$
$66 \wedge 103 \wedge 244 \wedge 253 \wedge 208 \wedge 4 \wedge 140 \wedge 2 \longrightarrow 277$
$5 \wedge 9 \longrightarrow 21$
$61 \wedge 165 \wedge 4 \wedge 147 \longrightarrow 211$
$41 \wedge 168 \wedge 96 \wedge 185 \wedge 54 \wedge 75 \longrightarrow 262$
$1 \wedge 15 \wedge 17 \longrightarrow 25$
$112 \wedge 130 \wedge 88 \wedge 27 \wedge 4 \longrightarrow 144$
$79 \wedge 27 \wedge 46 \wedge 31 \wedge 33 \longrightarrow 87$
$74 \wedge 13 \wedge 52 \wedge 53 \wedge 108 \wedge 50 \wedge 72 \longrightarrow 143$
$23 \wedge 3 \wedge 49 \wedge 8 \wedge 21 \wedge 25 \wedge 38 \wedge 54 \longrightarrow 65$
$11 \wedge 76 \wedge 40 \longrightarrow 217$
$2 \wedge 6 \wedge 29 \wedge 30 \wedge 16 \wedge 7 \longrightarrow 33$
$73 \wedge 96 \wedge 23 \wedge 4 \wedge 53 \longrightarrow 153$
$36 \wedge 208 \wedge 60 \wedge 112 \wedge 52 \wedge 136 \wedge 59 \wedge 235 \longrightarrow 262$
$40 \wedge 67 \wedge 119 \wedge 220 \wedge 57 \longrightarrow 234$
$209 \wedge 202 \wedge 188 \wedge 137 \wedge 134 \longrightarrow 276$
$38 \wedge 11 \wedge 103 \wedge 6 \wedge 73 \longrightarrow 122$
$6 \wedge 17 \wedge 14 \wedge 1 \wedge 13 \wedge 15 \wedge 9 \wedge 2 \longrightarrow 21$
$11 \wedge 14 \wedge 54 \wedge 18 \wedge 50 \wedge 36 \wedge 6 \wedge 1 \longrightarrow 63$
$53 \longrightarrow 135$
$19 \wedge 42 \wedge 16 \wedge 26 \wedge 8 \longrightarrow 43$
$1 \longrightarrow 2$

$72 \wedge 22 \wedge 40 \wedge 51 \wedge 44 \longrightarrow 108$
$50 \wedge 16 \wedge 100 \wedge 1 \wedge 59 \wedge 80 \longrightarrow 108$
$198 \wedge 141 \wedge 36 \wedge 228 \wedge 216 \wedge 115 \longrightarrow 246$
$160 \wedge 68 \longrightarrow 294$
$42 \wedge 16 \wedge 39 \wedge 41 \wedge 36 \wedge 5 \wedge 14 \longrightarrow 45$
$17 \wedge 26 \wedge 55 \wedge 243 \wedge 144 \wedge 122 \wedge 190 \wedge 135 \longrightarrow 293$
$9 \longrightarrow 118$
$162 \wedge 31 \longrightarrow 217$
$90 \longrightarrow 143$
$85 \wedge 161 \wedge 256 \wedge 90 \longrightarrow 293$
$7 \wedge 2 \wedge 1 \wedge 5 \wedge 4 \longrightarrow 8$
$112 \wedge 19 \wedge 86 \wedge 146 \wedge 61 \wedge 156 \wedge 53 \wedge 18 \longrightarrow 161$
$26 \wedge 168 \wedge 9 \wedge 117 \wedge 2 \wedge 174 \wedge 210 \longrightarrow 232$
$13 \wedge 74 \wedge 43 \wedge 2 \wedge 80 \wedge 23 \wedge 28 \wedge 89 \longrightarrow 101$
$35 \wedge 68 \wedge 65 \wedge 66 \wedge 43 \wedge 17 \longrightarrow 69$
$45 \wedge 8 \longrightarrow 65$
$77 \wedge 26 \wedge 23 \wedge 25 \longrightarrow 133$
$135 \wedge 11 \longrightarrow 188$
$1 \longrightarrow 2$
$10 \wedge 23 \wedge 18 \longrightarrow 33$
$21 \longrightarrow 66$
$8 \wedge 5 \wedge 10 \wedge 18 \wedge 14 \wedge 3 \wedge 12 \longrightarrow 21$
$128 \wedge 81 \wedge 76 \wedge 87 \wedge 96 \wedge 40 \wedge 56 \wedge 48 \longrightarrow 147$
$197 \wedge 113 \wedge 75 \wedge 146 \wedge 7 \wedge 91 \wedge 70 \longrightarrow 212$
$201 \wedge 204 \wedge 18 \wedge 235 \wedge 227 \wedge 141 \wedge 134 \longrightarrow 277$
$6 \wedge 5 \wedge 9 \wedge 8 \wedge 2 \longrightarrow 10$
$111 \wedge 54 \wedge 105 \longrightarrow 143$
$141 \wedge 244 \wedge 104 \wedge 45 \wedge 20 \wedge 225 \wedge 233 \wedge 44 \longrightarrow 247$
$46 \wedge 14 \wedge 31 \wedge 22 \wedge 34 \wedge 6 \wedge 27 \longrightarrow 49$
$143 \wedge 146 \wedge 154 \longrightarrow 269$
$31 \wedge 41 \wedge 15 \longrightarrow 49$
$181 \wedge 152 \wedge 53 \wedge 96 \wedge 120 \longrightarrow 187$
$213 \wedge 159 \longrightarrow 214$
$190 \wedge 135 \longrightarrow 211$
$23 \wedge 33 \longrightarrow 59$
$38 \wedge 32 \wedge 104 \wedge 110 \wedge 75 \longrightarrow 157$
$127 \wedge 63 \wedge 34 \wedge 62 \longrightarrow 133$
$172 \longrightarrow 227$
$166 \longrightarrow 226$
$29 \longrightarrow 33$
$10 \wedge 34 \longrightarrow 117$
$31 \wedge 139 \wedge 134 \wedge 115 \wedge 10 \longrightarrow 206$
$124 \wedge 184 \wedge 29 \wedge 146 \longrightarrow 234$
$7 \wedge 1 \wedge 6 \wedge 4 \wedge 3 \longrightarrow 8$

$27 \wedge 12 \wedge 73 \wedge 42 \wedge 28 \wedge 49 \wedge 66 \longrightarrow 77$
$6 \longrightarrow 182$
$6 \wedge 20 \wedge 4 \longrightarrow 21$
$115 \wedge 134 \wedge 99 \longrightarrow 147$
$11 \wedge 50 \wedge 35 \wedge 45 \longrightarrow 59$
$203 \wedge 37 \wedge 116 \longrightarrow 276$
$87 \wedge 103 \longrightarrow 147$
$64 \wedge 225 \wedge 203 \wedge 224 \longrightarrow 279$
$52 \wedge 80 \wedge 129 \wedge 94 \longrightarrow 224$
$33 \wedge 73 \wedge 90 \wedge 31 \longrightarrow 97$
$37 \wedge 43 \wedge 76 \wedge 178 \longrightarrow 270$
$51 \wedge 52 \wedge 143 \wedge 138 \wedge 91 \wedge 156 \wedge 39 \longrightarrow 188$
$36 \wedge 68 \wedge 52 \longrightarrow 161$
$103 \wedge 90 \longrightarrow 110$
$163 \wedge 74 \wedge 129 \longrightarrow 234$
$7 \wedge 6 \wedge 4 \longrightarrow 25$
$27 \longrightarrow 31$
$81 \wedge 72 \wedge 108 \wedge 88 \wedge 61 \longrightarrow 143$
$92 \wedge 18 \wedge 11 \wedge 89 \wedge 96 \wedge 108 \wedge 13 \wedge 8 \longrightarrow 113$
$55 \wedge 35 \wedge 38 \wedge 4 \longrightarrow 65$
$52 \wedge 96 \wedge 14 \wedge 116 \wedge 15 \wedge 25 \longrightarrow 119$
$39 \wedge 264 \wedge 66 \wedge 23 \wedge 98 \wedge 143 \wedge 147 \longrightarrow 273$
$8 \wedge 17 \wedge 33 \wedge 22 \wedge 47 \wedge 20 \longrightarrow 55$
$128 \wedge 127 \wedge 188 \wedge 211 \wedge 238 \wedge 114 \wedge 203 \longrightarrow 240$
$65 \longrightarrow 292$
$6 \wedge 19 \wedge 16 \longrightarrow 21$
$21 \wedge 24 \wedge 15 \wedge 28 \longrightarrow 73$
$63 \wedge 11 \wedge 14 \wedge 27 \wedge 65 \longrightarrow 92$
$26 \wedge 54 \wedge 133 \wedge 22 \longrightarrow 226$
$156 \wedge 185 \longrightarrow 245$
$7 \wedge 10 \longrightarrow 33$
$88 \longrightarrow 159$
$127 \wedge 128 \wedge 29 \longrightarrow 273$
$140 \wedge 83 \wedge 76 \wedge 122 \wedge 35 \wedge 170 \wedge 160 \longrightarrow 206$
$2 \wedge 64 \wedge 66 \wedge 1 \wedge 105 \longrightarrow 143$
$22 \wedge 41 \wedge 55 \wedge 51 \wedge 95 \wedge 105 \wedge 98 \longrightarrow 122$
$164 \wedge 152 \wedge 117 \wedge 94 \longrightarrow 214$
$13 \wedge 77 \wedge 14 \wedge 68 \wedge 62 \wedge 40 \wedge 10 \longrightarrow 101$
$35 \wedge 78 \wedge 29 \wedge 57 \longrightarrow 85$
$56 \wedge 3 \wedge 15 \wedge 54 \wedge 1 \wedge 57 \wedge 6 \wedge 2 \longrightarrow 69$
$54 \wedge 65 \wedge 103 \longrightarrow 110$
$37 \wedge 51 \wedge 29 \wedge 88 \wedge 50 \wedge 3 \longrightarrow 90$
$101 \wedge 25 \wedge 1 \wedge 36 \wedge 57 \wedge 71 \wedge 7 \longrightarrow 110$
$27 \wedge 38 \wedge 14 \wedge 16 \wedge 9 \wedge 25 \longrightarrow 49$

$52 \wedge 104 \longrightarrow 245$
$88 \wedge 146 \wedge 110 \longrightarrow 216$
$116 \wedge 22 \wedge 105 \longrightarrow 293$
$15 \wedge 19 \wedge 7 \wedge 2 \wedge 22 \longrightarrow 25$
$2 \wedge 4 \wedge 1 \longrightarrow 6$
$101 \wedge 103 \wedge 180 \wedge 33 \longrightarrow 211$
$1 \wedge 23 \wedge 64 \wedge 61 \wedge 29 \wedge 76 \wedge 20 \longrightarrow 156$
$13 \wedge 25 \wedge 66 \longrightarrow 217$
$54 \wedge 74 \wedge 76 \wedge 214 \longrightarrow 215$
$1 \wedge 2 \longrightarrow 6$
$49 \wedge 207 \wedge 155 \wedge 90 \wedge 41 \wedge 149 \longrightarrow 234$
$47 \wedge 68 \wedge 50 \wedge 19 \wedge 18 \wedge 36 \longrightarrow 69$
$242 \wedge 101 \wedge 227 \longrightarrow 247$
$17 \wedge 106 \wedge 124 \wedge 240 \wedge 123 \wedge 67 \wedge 10 \longrightarrow 245$
$18 \wedge 34 \wedge 51 \wedge 50 \wedge 55 \wedge 117 \longrightarrow 161$
$45 \wedge 65 \wedge 50 \wedge 23 \wedge 18 \wedge 1 \wedge 49 \longrightarrow 67$
$16 \wedge 102 \wedge 68 \wedge 289 \longrightarrow 293$
$35 \wedge 28 \wedge 20 \wedge 34 \wedge 14 \wedge 32 \longrightarrow 36$
$168 \wedge 67 \wedge 209 \wedge 94 \wedge 139 \wedge 1 \longrightarrow 230$
$56 \wedge 101 \longrightarrow 276$
$128 \wedge 280 \wedge 263 \wedge 195 \wedge 166 \wedge 179 \wedge 114 \longrightarrow 297$
$13 \longrightarrow 54$
$55 \wedge 69 \wedge 16 \wedge 85 \longrightarrow 96$
$22 \wedge 11 \wedge 15 \wedge 24 \wedge 20 \wedge 4 \longrightarrow 31$
$68 \wedge 42 \wedge 37 \wedge 211 \wedge 21 \wedge 108 \longrightarrow 217$
$1 \longrightarrow 2$
$17 \wedge 8 \wedge 25 \wedge 26 \wedge 1 \wedge 20 \wedge 16 \wedge 5 \longrightarrow 33$
$59 \wedge 14 \longrightarrow 232$
$32 \wedge 59 \wedge 40 \wedge 53 \wedge 8 \wedge 23 \wedge 22 \longrightarrow 69$
$1 \wedge 90 \longrightarrow 134$
$54 \wedge 89 \wedge 64 \wedge 94 \wedge 14 \wedge 66 \longrightarrow 102$
$146 \longrightarrow 276$
$54 \wedge 130 \wedge 235 \wedge 198 \wedge 35 \wedge 114 \longrightarrow 279$
$21 \wedge 5 \wedge 70 \wedge 82 \longrightarrow 118$
$182 \longrightarrow 279$
$16 \wedge 1 \wedge 27 \wedge 26 \wedge 36 \wedge 10 \wedge 15 \wedge 2 \longrightarrow 39$
$230 \wedge 194 \longrightarrow 267$
$54 \wedge 81 \wedge 100 \wedge 134 \wedge 127 \wedge 14 \longrightarrow 216$
$20 \wedge 114 \wedge 112 \longrightarrow 135$
$24 \wedge 68 \wedge 171 \wedge 131 \wedge 1 \longrightarrow 211$
$188 \wedge 233 \wedge 259 \longrightarrow 293$
$155 \wedge 242 \wedge 81 \longrightarrow 246$
$47 \wedge 183 \longrightarrow 277$
$23 \wedge 165 \wedge 249 \wedge 129 \wedge 215 \longrightarrow 273$

$101 \wedge 93 \wedge 40 \wedge 89 \wedge 265 \longrightarrow 269$
$20 \wedge 1 \longrightarrow 34$
$4 \wedge 142 \wedge 100 \wedge 165 \longrightarrow 199$
$173 \wedge 152 \wedge 75 \wedge 180 \wedge 101 \wedge 72 \wedge 94 \wedge 82 \longrightarrow 293$
$198 \wedge 11 \wedge 50 \wedge 69 \wedge 101 \wedge 18 \longrightarrow 271$
$29 \wedge 12 \longrightarrow 55$
$107 \wedge 110 \wedge 65 \wedge 139 \wedge 178 \wedge 58 \longrightarrow 192$
$193 \longrightarrow 221$
$23 \wedge 85 \wedge 160 \wedge 150 \longrightarrow 227$
$208 \wedge 5 \wedge 70 \longrightarrow 245$
$47 \wedge 53 \wedge 57 \wedge 17 \wedge 16 \wedge 36 \wedge 10 \longrightarrow 63$
$180 \wedge 88 \longrightarrow 181$
$104 \wedge 164 \wedge 64 \longrightarrow 173$
$162 \wedge 20 \wedge 62 \wedge 61 \wedge 21 \wedge 87 \wedge 102 \longrightarrow 174$
$3 \wedge 22 \wedge 4 \wedge 18 \wedge 31 \wedge 8 \longrightarrow 45$
$193 \wedge 124 \wedge 186 \longrightarrow 264$
$20 \longrightarrow 297$
$236 \wedge 150 \wedge 139 \wedge 213 \longrightarrow 240$
$84 \wedge 60 \wedge 103 \longrightarrow 293$
$12 \wedge 201 \wedge 173 \wedge 179 \wedge 137 \wedge 180 \wedge 51 \longrightarrow 211$
$37 \wedge 22 \wedge 3 \wedge 138 \wedge 94 \wedge 128 \longrightarrow 224$
$157 \wedge 96 \wedge 257 \wedge 174 \longrightarrow 279$
$31 \wedge 35 \wedge 79 \wedge 62 \wedge 153 \wedge 91 \wedge 209 \longrightarrow 226$
$88 \wedge 100 \wedge 47 \wedge 113 \wedge 30 \wedge 18 \wedge 61 \longrightarrow 166$
$2 \longrightarrow 4$
$51 \wedge 77 \wedge 124 \wedge 169 \longrightarrow 171$
$129 \wedge 60 \wedge 11 \wedge 12 \wedge 65 \wedge 28 \longrightarrow 135$
$12 \wedge 15 \wedge 1 \longrightarrow 17$
$98 \wedge 73 \wedge 9 \wedge 38 \wedge 100 \wedge 94 \wedge 24 \longrightarrow 113$
$245 \wedge 97 \wedge 35 \wedge 82 \wedge 213 \wedge 81 \longrightarrow 294$
$34 \wedge 45 \longrightarrow 49$
$20 \wedge 31 \wedge 190 \wedge 141 \wedge 123 \longrightarrow 292$
$55 \wedge 110 \wedge 54 \longrightarrow 148$
$119 \wedge 20 \wedge 188 \wedge 133 \wedge 122 \longrightarrow 216$
$66 \wedge 55 \wedge 98 \longrightarrow 110$
$50 \longrightarrow 55$
$10 \wedge 19 \wedge 23 \wedge 3 \wedge 9 \wedge 4 \longrightarrow 45$
$161 \wedge 150 \wedge 160 \wedge 236 \wedge 34 \wedge 225 \longrightarrow 237$
$54 \wedge 111 \wedge 2 \wedge 6 \wedge 146 \wedge 32 \wedge 1 \wedge 140 \longrightarrow 148$
$175 \wedge 58 \wedge 17 \wedge 4 \wedge 75 \wedge 65 \wedge 23 \longrightarrow 192$
$55 \longrightarrow 217$
$1 \longrightarrow 2$
$43 \wedge 4 \wedge 7 \longrightarrow 117$
$194 \wedge 129 \wedge 6 \wedge 198 \wedge 230 \wedge 21 \wedge 45 \longrightarrow 251$

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38 ⋀ 44 ⋀ 145 ⋀ 204 ⟶ 266
75 ⋀ 35 ⟶ 92
215 ⋀ 67 ⟶ 230
148 ⋀ 51 ⋀ 30 ⋀ 119 ⋀ 97 ⋀ 12 ⋀ 48 ⋀ 183 ⟶212
14 ⟶ 21
42 ⋀ 27 ⟶ 43
66 ⋀ 93 ⟶ 108
16 ⋀ 80 ⋀ 177 ⋀ 195 ⟶ 273
109 ⋀ 101 ⋀ 68 ⟶ 113
49 ⋀ 8 ⋀ 22 ⋀ 107 ⋀ 190 ⋀ 178 ⟶ 194
5 ⋀ 8 ⋀ 6 ⋀ 4 ⋀ 1 ⋀ 3 ⟶ 14
128 ⋀ 18 ⋀ 132 ⋀ 250 ⋀ 175 ⋀ 33 ⋀ 111 ⟶ 253
256 ⋀ 158 ⋀ 227 ⋀ 188 ⋀ 154 ⋀ 18 ⋀ 37 ⋀ 203 ⟶267
80 ⋀ 66 ⋀ 84 ⋀ 81 ⋀ 13 ⋀ 54 ⟶ 188
72 ⋀ 36 ⋀ 43 ⋀ 132 ⋀ 80 ⟶ 133
151 ⋀ 44 ⋀ 216 ⋀ 11 ⋀ 165 ⋀ 18 ⟶ 269
68 ⟶ 85
67 ⋀ 17 ⋀ 16 ⋀ 122 ⋀ 42 ⟶ 157
69 ⋀ 2 ⋀ 5 ⟶ 87
193 ⋀ 178 ⋀ 136 ⟶ 267
13 ⋀ 26 ⋀ 35 ⋀ 39 ⋀ 37 ⋀ 27 ⋀ 12 ⋀ 33 ⟶45
185 ⋀ 131 ⋀ 165 ⋀ 116 ⋀ 177 ⋀ 16 ⋀ 184 ⟶ 200
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188 ⋀ 95 ⋀ 117 ⋀ 202 ⋀ 113 ⋀ 3 ⋀ 61 ⋀ 206 ⟶253
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28 ⋀ 17 ⋀ 22 ⋀ 12 ⋀ 8 ⟶ 36
36 ⋀ 60 ⋀ 51 ⋀ 57 ⋀ 23 ⟶ 96
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82 ⋀ 119 ⋀ 51 ⋀ 58 ⋀ 33 ⋀ 9 ⋀ 49 ⟶ 143
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138 ⋀ 68 ⋀ 107 ⋀ 81 ⋀ 136 ⋀ 14 ⋀ 125 ⋀ 94 ⟶200
31 ⋀ 7 ⋀ 27 ⋀ 117 ⋀ 110 ⋀ 6 ⋀ 37 ⋀ 47 ⟶122
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3 ⋀ 1 ⟶ 4
167 ⋀ 30 ⋀ 221 ⋀ 198 ⟶ 226
129 ⋀ 45 ⋀ 5 ⋀ 74 ⋀ 104 ⋀ 68 ⟶ 144
2 ⋀ 3 ⋀ 1 ⟶ 4
23 ⋀ 105 ⋀ 93 ⋀ 31 ⋀ 119 ⟶ 224
130 ⋀ 1 ⋀ 119 ⋀ 124 ⋀ 105 ⋀ 33 ⋀ 260 ⟶ 296
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$24 \wedge 48 \wedge 25 \wedge 51 \wedge 2 \longrightarrow 55$
$3 \wedge 7 \wedge 5 \wedge 1 \wedge 2 \wedge 4 \longrightarrow 8$
$91 \wedge 97 \wedge 4 \wedge 86 \wedge 132 \wedge 128 \wedge 52 \longrightarrow 133$
$108 \wedge 206 \longrightarrow 214$
$167 \wedge 52 \wedge 132 \wedge 184 \wedge 124 \longrightarrow 186$
$141 \wedge 75 \wedge 110 \wedge 126 \longrightarrow 173$
$3 \wedge 15 \wedge 32 \longrightarrow 146$
$39 \wedge 20 \longrightarrow 49$
$32 \wedge 5 \wedge 61 \wedge 22 \longrightarrow 65$
$7 \wedge 147 \wedge 39 \wedge 59 \wedge 94 \wedge 80 \wedge 74 \longrightarrow 264$
$86 \wedge 37 \wedge 45 \longrightarrow 269$
$52 \wedge 50 \wedge 13 \wedge 6 \wedge 12 \wedge 2 \wedge 7 \longrightarrow 54$
$37 \longrightarrow 65$
$32 \wedge 38 \wedge 30 \wedge 57 \wedge 20 \wedge 7 \wedge 13 \longrightarrow 66$
$21 \wedge 125 \wedge 11 \wedge 156 \wedge 120 \longrightarrow 176$
$23 \wedge 73 \wedge 5 \wedge 196 \wedge 79 \wedge 101 \wedge 199 \wedge 128 \longrightarrow 216$
$89 \wedge 149 \wedge 237 \wedge 65 \wedge 143 \wedge 269 \wedge 171 \wedge 222 \longrightarrow 279$
$151 \wedge 139 \longrightarrow 199$
$118 \wedge 103 \wedge 217 \longrightarrow 245$
$49 \wedge 8 \wedge 42 \wedge 52 \wedge 58 \longrightarrow 77$
$2 \longrightarrow 66$
$127 \wedge 172 \wedge 10 \wedge 197 \wedge 64 \wedge 135 \wedge 169 \wedge 163 \longrightarrow 227$
$243 \wedge 113 \wedge 172 \wedge 86 \longrightarrow 292$
$89 \wedge 157 \wedge 9 \wedge 105 \wedge 84 \wedge 51 \wedge 223 \wedge 251 \longrightarrow 266$
$4 \wedge 3 \wedge 1 \wedge 2 \wedge 5 \longrightarrow 6$
$55 \wedge 62 \wedge 25 \wedge 91 \wedge 32 \longrightarrow 98$
$88 \longrightarrow 161$
$62 \wedge 139 \wedge 58 \wedge 108 \longrightarrow 229$
$19 \wedge 24 \wedge 9 \wedge 13 \wedge 11 \wedge 21 \wedge 23 \wedge 28 \longrightarrow 33$
$38 \wedge 81 \wedge 128 \wedge 17 \wedge 125 \longrightarrow 143$
$58 \wedge 87 \wedge 52 \longrightarrow 134$
$1 \longrightarrow 4$
$34 \wedge 224 \wedge 44 \wedge 257 \wedge 114 \wedge 186 \longrightarrow 277$
$95 \wedge 90 \wedge 178 \longrightarrow 267$
$29 \wedge 14 \wedge 22 \wedge 18 \wedge 26 \wedge 37 \wedge 2 \wedge 19 \longrightarrow 43$
$50 \wedge 38 \wedge 5 \wedge 230 \wedge 111 \wedge 121 \wedge 229 \longrightarrow 232$
$60 \wedge 173 \wedge 33 \wedge 122 \wedge 186 \wedge 212 \wedge 185 \longrightarrow 219$
$80 \wedge 172 \wedge 15 \longrightarrow 232$
$133 \wedge 19 \longrightarrow 143$
$44 \wedge 89 \wedge 38 \wedge 36 \wedge 41 \wedge 29 \wedge 58 \longrightarrow 102$
$50 \wedge 33 \wedge 37 \longrightarrow 66$
$66 \longrightarrow 181$
$56 \wedge 50 \wedge 54 \longrightarrow 117$
$26 \wedge 19 \wedge 98 \wedge 211 \wedge 1 \wedge 87 \wedge 153 \longrightarrow 246$

155 ∧ 63 ∧ 1 ∧ 94 ∧ 103 ∧ 34 ∧ 68 ∧ 55 ⟶182
62 ∧ 116 ∧ 123 ∧ 84 ∧ 25 ∧ 67 ⟶ 182
127 ∧ 4 ∧ 154 ∧ 98 ∧ 146 ∧ 126 ∧ 151 ∧ 51 ⟶166
48 ∧ 50 ∧ 46 ∧ 51 ∧ 97 ⟶ 101
33 ∧ 74 ⟶ 96
131 ∧ 76 ⟶ 181
157 ∧ 174 ∧ 124 ⟶ 188
152 ∧ 62 ∧ 214 ∧ 185 ∧ 215 ∧ 136 ∧ 201 ⟶ 227
4 ∧ 265 ∧ 112 ∧ 139 ∧ 235 ∧ 184 ⟶ 297
80 ∧ 34 ∧ 192 ∧ 23 ⟶ 230
261 ∧ 169 ∧ 152 ∧ 89 ∧ 33 ∧ 206 ∧ 216 ⟶ 262
42 ∧ 66 ∧ 37 ∧ 47 ∧ 76 ⟶ 97
133 ∧ 216 ∧ 84 ∧ 111 ∧ 258 ∧ 251 ⟶ 271
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82 ∧ 219 ∧ 164 ∧ 59 ∧ 7 ∧ 30 ∧ 153 ∧ 105 ⟶273
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27 ∧ 13 ∧ 19 ∧ 11 ∧ 14 ⟶ 45
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3 ∧ 10 ⟶ 14
1 ∧ 6 ∧ 3 ∧ 5 ∧ 4 ⟶ 8
5 ∧ 32 ∧ 27 ∧ 7 ∧ 1 ∧ 22 ⟶ 39
28 ∧ 13 ⟶ 217
145 ∧ 201 ∧ 7 ∧ 163 ∧ 172 ∧ 189 ∧ 68 ∧ 49 ⟶215
121 ∧ 165 ⟶ 199
26 ∧ 182 ∧ 127 ⟶ 224
156 ∧ 102 ∧ 186 ∧ 173 ⟶ 279
19 ∧ 26 ⟶ 31
134 ∧ 51 ⟶ 186
148 ∧ 26 ∧ 124 ∧ 91 ∧ 93 ⟶ 168
2 ∧ 5 ⟶ 17
268 ∧ 35 ∧ 96 ∧ 175 ∧ 179 ∧ 21 ⟶ 270
79 ⟶ 148
38 ∧ 110 ∧ 135 ∧ 11 ∧ 140 ∧ 98 ⟶ 181
107 ∧ 230 ∧ 8 ∧ 77 ⟶ 237
110 ∧ 23 ∧ 15 ∧ 204 ∧ 181 ⟶ 206
146 ∧ 31 ∧ 276 ∧ 227 ∧ 111 ∧ 140 ∧ 201 ⟶ 297
130 ∧ 217 ∧ 57 ⟶ 297
101 ∧ 66 ∧ 153 ∧ 105 ∧ 43 ∧ 14 ∧ 137 ∧ 65 ⟶186
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2 ∧ 3 ∧ 6 ∧ 4 ∧ 5 ∧ 1 ∧ 7 ⟶ 8

36 ∧ 83 ∧ 52 ∧ 109 ∧ 96 ∧ 43 ∧ 1 ∧ 106 ⟶119
52 ∧ 5 ∧ 43 ∧ 49 ∧ 1 ⟶ 55
3 ∧ 5 ⟶ 6
13 ∧ 172 ⟶ 227
34 ∧ 4 ⟶ 85
2 ∧ 1 ∧ 5 ∧ 3 ⟶ 10
173 ⟶ 221
238 ∧ 22 ∧ 90 ∧ 191 ⟶ 262
137 ∧ 129 ∧ 30 ∧ 121 ∧ 138 ⟶ 277
6 ∧ 207 ∧ 64 ∧ 233 ∧ 96 ∧ 268 ⟶ 297
18 ∧ 61 ∧ 5 ∧ 35 ∧ 50 ∧ 77 ∧ 13 ∧ 125 ⟶133
124 ∧ 212 ∧ 15 ∧ 20 ∧ 175 ∧ 148 ∧ 42 ⟶ 246
108 ∧ 87 ∧ 48 ⟶ 122
140 ∧ 14 ∧ 5 ⟶ 157
107 ∧ 35 ∧ 77 ∧ 50 ∧ 138 ∧ 98 ∧ 127 ∧ 53 ⟶168
95 ∧ 39 ∧ 13 ∧ 221 ∧ 201 ∧ 228 ⟶ 245
51 ∧ 34 ∧ 141 ∧ 71 ∧ 45 ∧ 118 ∧ 63 ∧ 59 ⟶176
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3 ∧ 54 ∧ 28 ∧ 18 ∧ 48 ∧ 67 ∧ 87 ⟶ 92
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137 ∧ 144 ∧ 4 ∧ 15 ∧ 217 ∧ 6 ∧ 211 ∧ 149 ⟶229
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5 ∧ 8 ∧ 11 ∧ 4 ⟶ 14
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70 ∧ 44 ∧ 54 ∧ 36 ∧ 57 ∧ 35 ∧ 53 ⟶ 77
20 ∧ 36 ∧ 13 ∧ 16 ∧ 2 ∧ 14 ⟶ 39
77 ∧ 102 ∧ 66 ∧ 65 ∧ 19 ∧ 30 ∧ 86 ∧ 15 ⟶113
2 ⟶ 6
137 ∧ 129 ∧ 175 ∧ 155 ∧ 97 ∧ 156 ⟶ 188
139 ∧ 181 ∧ 153 ∧ 112 ∧ 183 ∧ 167 ∧ 238 ∧ 265 ⟶276

$57 \wedge 48 \wedge 45 \wedge 36 \wedge 62 \longrightarrow 108$
$52 \wedge 89 \wedge 46 \wedge 35 \wedge 15 \wedge 91 \longrightarrow 111$
$283 \wedge 230 \wedge 117 \longrightarrow 292$
$87 \wedge 43 \wedge 50 \wedge 71 \longrightarrow 98$
$17 \longrightarrow 45$
$127 \wedge 72 \wedge 48 \wedge 116 \wedge 198 \wedge 125 \longrightarrow 217$
$48 \longrightarrow 182$
$23 \wedge 36 \wedge 7 \wedge 49 \longrightarrow 73$
$10 \wedge 93 \longrightarrow 108$
$206 \longrightarrow 237$
$104 \longrightarrow 156$
$158 \wedge 28 \wedge 75 \wedge 85 \wedge 208 \wedge 126 \longrightarrow 211$
$28 \wedge 205 \wedge 35 \wedge 121 \wedge 179 \wedge 115 \wedge 93 \wedge 178 \longrightarrow 217$
$39 \wedge 7 \wedge 19 \wedge 47 \longrightarrow 62$
$122 \wedge 24 \wedge 39 \longrightarrow 212$
$217 \wedge 262 \wedge 128 \longrightarrow 277$
$172 \wedge 122 \wedge 100 \wedge 164 \wedge 128 \longrightarrow 227$
$194 \wedge 208 \wedge 110 \wedge 98 \wedge 187 \wedge 21 \wedge 97 \wedge 95 \longrightarrow 229$
$276 \wedge 249 \longrightarrow 277$
$226 \wedge 12 \wedge 116 \wedge 54 \wedge 231 \longrightarrow 251$
$128 \wedge 81 \wedge 92 \wedge 146 \wedge 98 \wedge 94 \wedge 76 \wedge 158 \longrightarrow 159$
$24 \wedge 23 \wedge 112 \wedge 46 \wedge 129 \wedge 36 \wedge 28 \wedge 59 \longrightarrow 135$
$21 \wedge 28 \wedge 38 \wedge 19 \wedge 15 \longrightarrow 59$
$11 \wedge 7 \wedge 27 \wedge 61 \wedge 35 \longrightarrow 73$
$16 \wedge 18 \wedge 13 \wedge 5 \wedge 19 \wedge 4 \longrightarrow 25$
$157 \wedge 3 \wedge 206 \longrightarrow 226$
$146 \wedge 15 \wedge 118 \longrightarrow 153$
$59 \wedge 67 \wedge 93 \wedge 41 \longrightarrow 112$
$103 \wedge 40 \wedge 61 \longrightarrow 166$
$178 \wedge 33 \wedge 76 \wedge 31 \wedge 192 \wedge 212 \wedge 242 \wedge 238 \longrightarrow 248$
$21 \wedge 76 \wedge 61 \wedge 51 \longrightarrow 119$
$38 \wedge 124 \wedge 53 \wedge 29 \longrightarrow 168$
$95 \wedge 39 \wedge 18 \wedge 9 \wedge 103 \wedge 29 \wedge 30 \wedge 89 \longrightarrow 108$
$143 \wedge 94 \longrightarrow 156$
$36 \wedge 2 \wedge 25 \wedge 11 \wedge 32 \wedge 10 \wedge 7 \wedge 18 \longrightarrow 49$
$48 \wedge 27 \wedge 20 \wedge 42 \longrightarrow 62$
$139 \wedge 149 \wedge 33 \wedge 23 \wedge 55 \wedge 1 \wedge 67 \wedge 115 \longrightarrow 199$
$92 \wedge 11 \wedge 258 \wedge 81 \wedge 9 \longrightarrow 270$
$23 \wedge 31 \wedge 51 \longrightarrow 65$
$132 \wedge 102 \wedge 24 \wedge 59 \wedge 26 \wedge 23 \wedge 11 \longrightarrow 146$
$78 \wedge 41 \longrightarrow 143$
$152 \wedge 78 \wedge 12 \longrightarrow 296$
$118 \wedge 78 \wedge 3 \wedge 186 \longrightarrow 230$
$28 \wedge 136 \wedge 135 \wedge 137 \wedge 110 \wedge 83 \longrightarrow 220$

$1 \wedge 3 \longrightarrow 8$
$167 \wedge 46 \wedge 149 \wedge 151 \wedge 137 \longrightarrow 214$
$47 \longrightarrow 118$
$73 \wedge 19 \wedge 24 \wedge 85 \wedge 10 \wedge 5 \longrightarrow 92$
$17 \wedge 33 \wedge 80 \wedge 151 \longrightarrow 168$
$55 \wedge 60 \wedge 48 \wedge 33 \wedge 11 \wedge 43 \longrightarrow 87$
$129 \wedge 106 \wedge 103 \wedge 41 \wedge 53 \longrightarrow 215$
$39 \wedge 112 \wedge 36 \wedge 196 \wedge 195 \wedge 33 \longrightarrow 251$
$174 \longrightarrow 270$
$21 \wedge 1 \wedge 80 \wedge 47 \wedge 59 \wedge 68 \wedge 12 \wedge 70 \longrightarrow 92$
$39 \wedge 87 \wedge 78 \wedge 52 \wedge 45 \wedge 119 \wedge 145 \wedge 121 \longrightarrow 161$
$104 \wedge 13 \wedge 74 \wedge 61 \wedge 11 \wedge 33 \wedge 40 \wedge 67 \longrightarrow 110$
$50 \wedge 23 \wedge 26 \wedge 10 \wedge 19 \longrightarrow 63$
$11 \wedge 67 \wedge 102 \wedge 78 \wedge 101 \wedge 81 \wedge 16 \wedge 118 \longrightarrow 153$
$109 \wedge 158 \longrightarrow 227$
$24 \wedge 12 \longrightarrow 36$
$47 \longrightarrow 67$
$42 \wedge 34 \wedge 62 \wedge 68 \wedge 76 \wedge 27 \longrightarrow 96$
$169 \wedge 84 \wedge 208 \wedge 131 \wedge 174 \wedge 88 \wedge 4 \longrightarrow 214$
$68 \wedge 136 \wedge 105 \wedge 83 \wedge 84 \wedge 243 \longrightarrow 251$
$3 \wedge 2 \wedge 9 \longrightarrow 17$
$3 \wedge 18 \wedge 20 \wedge 31 \wedge 5 \wedge 21 \wedge 7 \longrightarrow 39$
$126 \wedge 31 \wedge 17 \wedge 76 \wedge 98 \wedge 153 \wedge 78 \longrightarrow 188$
$129 \wedge 140 \wedge 15 \wedge 20 \wedge 116 \longrightarrow 155$
$211 \wedge 225 \wedge 60 \wedge 15 \wedge 160 \longrightarrow 237$
$21 \wedge 89 \longrightarrow 135$
$47 \wedge 86 \wedge 25 \wedge 95 \wedge 64 \wedge 26 \longrightarrow 133$
$16 \wedge 22 \wedge 45 \wedge 29 \wedge 37 \wedge 43 \wedge 65 \longrightarrow 67$
$162 \wedge 107 \wedge 52 \wedge 38 \wedge 121 \wedge 62 \longrightarrow 230$
$98 \wedge 48 \wedge 85 \wedge 86 \wedge 43 \wedge 151 \wedge 25 \wedge 63 \longrightarrow 166$
$18 \wedge 116 \wedge 51 \wedge 68 \wedge 56 \wedge 59 \wedge 29 \wedge 8 \longrightarrow 118$
$65 \wedge 66 \wedge 4 \wedge 207 \wedge 121 \longrightarrow 215$
$62 \wedge 27 \wedge 20 \wedge 117 \wedge 54 \wedge 88 \wedge 70 \wedge 105 \longrightarrow 157$
$5 \wedge 2 \wedge 4 \longrightarrow 6$
$96 \wedge 103 \wedge 38 \wedge 173 \longrightarrow 182$
$77 \wedge 175 \wedge 83 \wedge 123 \wedge 79 \wedge 8 \wedge 114 \longrightarrow 269$
$74 \wedge 107 \wedge 93 \wedge 80 \wedge 57 \wedge 64 \longrightarrow 108$
$4 \wedge 1 \wedge 3 \wedge 2 \longrightarrow 6$
$99 \wedge 149 \wedge 126 \wedge 60 \wedge 78 \longrightarrow 161$
$61 \wedge 11 \wedge 37 \longrightarrow 65$
$213 \wedge 60 \wedge 137 \longrightarrow 276$
$62 \wedge 34 \wedge 60 \wedge 7 \longrightarrow 102$
$58 \wedge 18 \wedge 10 \wedge 54 \wedge 28 \wedge 33 \wedge 86 \longrightarrow 92$
$18 \longrightarrow 25$

77 ∧ 85 ∧ 70 ∧ 12 ∧ 86 ∧ 53 ∧ 21 ⟶ 97
44 ∧ 69 ∧ 72 ⟶ 73
11 ∧ 23 ∧ 91 ⟶ 96
34 ∧ 8 ∧ 201 ∧ 89 ∧ 213 ∧ 267 ∧ 108 ⟶ 271
188 ∧ 103 ∧ 79 ∧ 47 ∧ 153 ∧ 21 ∧ 99 ∧ 75 ⟶229
48 ∧ 2 ∧ 16 ∧ 23 ⟶ 55
64 ∧ 40 ∧ 105 ⟶ 117
78 ∧ 50 ∧ 59 ∧ 6 ∧ 90 ∧ 101 ⟶ 148
20 ⟶ 108
184 ∧ 108 ∧ 172 ∧ 29 ∧ 149 ∧ 44 ∧ 73 ∧ 152 ⟶248
28 ∧ 176 ∧ 149 ∧ 52 ∧ 77 ∧ 92 ∧ 127 ∧ 112 ⟶186
260 ∧ 90 ∧ 18 ∧ 273 ∧ 191 ⟶ 299
24 ∧ 157 ∧ 31 ⟶ 174
26 ∧ 44 ∧ 61 ∧ 96 ∧ 19 ⟶ 200
175 ∧ 197 ∧ 75 ∧ 193 ∧ 160 ∧ 74 ⟶ 266
46 ∧ 8 ⟶ 54
28 ∧ 17 ∧ 9 ∧ 19 ⟶ 34
28 ∧ 120 ∧ 123 ∧ 33 ∧ 95 ⟶ 146
249 ∧ 178 ∧ 57 ∧ 55 ∧ 189 ∧ 251 ∧ 59 ∧ 186 ⟶269
128 ∧ 32 ⟶ 273
12 ∧ 67 ∧ 33 ∧ 101 ∧ 45 ∧ 42 ∧ 76 ⟶ 117
121 ∧ 77 ∧ 111 ∧ 48 ∧ 118 ⟶ 143
65 ⟶ 116
121 ⟶ 146
85 ∧ 153 ∧ 131 ∧ 21 ∧ 1 ∧ 48 ∧ 83 ⟶ 176
112 ∧ 67 ∧ 90 ∧ 44 ∧ 51 ∧ 138 ⟶ 200
211 ∧ 16 ∧ 63 ∧ 184 ∧ 112 ∧ 208 ⟶ 220
62 ∧ 140 ∧ 194 ∧ 109 ⟶ 221
2 ∧ 62 ∧ 33 ∧ 42 ∧ 1 ∧ 22 ∧ 50 ∧ 87 ⟶96
111 ∧ 51 ∧ 26 ⟶ 134
275 ∧ 214 ∧ 12 ∧ 240 ∧ 76 ∧ 165 ∧ 167 ∧ 150 ⟶276
254 ∧ 148 ∧ 232 ⟶ 296
3 ∧ 107 ∧ 72 ⟶ 108
51 ∧ 133 ∧ 179 ∧ 119 ⟶ 212
32 ∧ 14 ⟶ 43
6 ⟶ 73
88 ∧ 214 ∧ 257 ∧ 191 ⟶ 266
181 ∧ 145 ∧ 26 ∧ 118 ∧ 126 ∧ 47 ∧ 184 ⟶ 219
10 ∧ 11 ∧ 59 ∧ 53 ⟶ 87
49 ∧ 28 ∧ 10 ∧ 69 ∧ 113 ⟶ 155
159 ∧ 75 ∧ 121 ∧ 140 ∧ 161 ∧ 119 ⟶ 216
74 ∧ 3 ∧ 77 ⟶ 195
35 ∧ 69 ∧ 48 ⟶ 87
138 ∧ 131 ∧ 149 ∧ 18 ∧ 38 ∧ 25 ∧ 96 ⟶ 219

85 ∧ 50 ∧ 31 ∧ 57 ∧ 53 ⟶ 101
127 ∧ 70 ⟶ 171
60 ∧ 58 ∧ 61 ∧ 45 ∧ 80 ∧ 117 ∧ 77 ⟶ 153
99 ∧ 8 ∧ 185 ∧ 135 ∧ 36 ∧ 149 ⟶ 229
6 ∧ 144 ∧ 97 ∧ 222 ∧ 126 ∧ 69 ∧ 155 ⟶ 234
10 ∧ 13 ∧ 5 ∧ 3 ⟶ 14
90 ∧ 29 ∧ 89 ⟶ 92
38 ⟶ 217
15 ∧ 231 ∧ 146 ∧ 6 ∧ 147 ∧ 216 ⟶ 240
10 ∧ 16 ⟶ 148
72 ∧ 38 ∧ 9 ∧ 58 ∧ 110 ∧ 6 ∧ 115 ⟶ 118
43 ∧ 68 ⟶ 148
94 ∧ 26 ⟶ 147
29 ∧ 22 ∧ 91 ∧ 27 ∧ 2 ∧ 104 ⟶ 135
5 ∧ 65 ⟶ 214
61 ∧ 4 ∧ 64 ∧ 88 ∧ 1 ∧ 85 ⟶ 92
18 ∧ 29 ∧ 20 ∧ 23 ∧ 25 ∧ 27 ⟶ 67
31 ⟶ 267
36 ⟶ 232
30 ∧ 142 ∧ 227 ∧ 54 ∧ 194 ∧ 120 ∧ 86 ⟶ 240
84 ∧ 165 ∧ 119 ∧ 218 ∧ 124 ⟶ 279
75 ∧ 8 ∧ 66 ⟶ 271
255 ∧ 263 ∧ 230 ⟶ 279
49 ∧ 70 ⟶ 144
29 ∧ 2 ∧ 73 ∧ 83 ⟶ 92
77 ∧ 93 ∧ 215 ∧ 80 ∧ 83 ∧ 22 ∧ 100 ⟶ 224
4 ⟶ 39
49 ∧ 154 ∧ 94 ⟶ 247
12 ⟶ 133
15 ⟶ 168
56 ⟶ 66
6 ∧ 78 ∧ 55 ∧ 70 ∧ 176 ∧ 149 ∧ 21 ⟶ 230
23 ∧ 28 ∧ 25 ∧ 16 ∧ 8 ⟶ 31
170 ⟶ 292
248 ∧ 31 ∧ 178 ∧ 21 ∧ 40 ⟶ 253
50 ∧ 168 ∧ 42 ∧ 84 ∧ 131 ⟶ 174
3 ∧ 60 ∧ 173 ∧ 169 ∧ 196 ∧ 25 ⟶ 226
55 ∧ 119 ∧ 90 ∧ 218 ∧ 152 ∧ 3 ∧ 173 ∧ 117 ⟶293
25 ⟶ 43
94 ∧ 84 ∧ 134 ∧ 155 ∧ 136 ⟶ 159
8 ∧ 4 ∧ 5 ∧ 2 ∧ 12 ∧ 1 ⟶ 17
173 ∧ 185 ∧ 63 ∧ 17 ∧ 118 ∧ 169 ⟶ 220
26 ∧ 127 ∧ 232 ∧ 168 ∧ 138 ⟶ 262
32 ∧ 64 ∧ 98 ∧ 103 ⟶ 133

65 ∧ 100 ∧ 4 ⟶ 101
5 ⟶ 21
152 ∧ 75 ∧ 94 ∧ 109 ∧ 81 ∧ 68 ∧ 143 ⟶ 153
178 ⟶ 192
12 ∧ 67 ∧ 50 ∧ 133 ⟶ 215
73 ∧ 80 ⟶ 118
27 ∧ 20 ∧ 36 ⟶ 69
292 ∧ 14 ∧ 88 ∧ 192 ∧ 146 ⟶ 296
106 ∧ 98 ∧ 120 ∧ 29 ∧ 110 ∧ 8 ∧ 76 ⟶ 194
132 ∧ 8 ∧ 84 ∧ 76 ⟶ 144
103 ∧ 106 ∧ 54 ∧ 70 ∧ 21 ⟶ 110
3 ∧ 20 ∧ 12 ∧ 1 ∧ 7 ∧ 19 ∧ 6 ∧ 15 ⟶25
1 ⟶ 2
6 ⟶ 39
8 ∧ 17 ∧ 16 ⟶ 117
148 ∧ 178 ∧ 41 ∧ 86 ⟶ 200
5 ∧ 4 ∧ 3 ⟶ 10
99 ∧ 198 ∧ 45 ∧ 64 ⟶ 229
98 ⟶ 226
103 ∧ 66 ∧ 124 ∧ 175 ∧ 11 ⟶ 227
26 ∧ 5 ∧ 48 ∧ 42 ∧ 4 ∧ 6 ⟶ 55
229 ∧ 208 ∧ 206 ∧ 168 ⟶ 273
249 ⟶ 273
64 ∧ 11 ∧ 20 ∧ 49 ∧ 87 ∧ 54 ⟶ 101
146 ∧ 45 ∧ 180 ∧ 76 ∧ 116 ∧ 204 ⟶ 253
4 ∧ 8 ∧ 13 ∧ 24 ∧ 23 ∧ 5 ∧ 7 ⟶ 25
35 ∧ 6 ∧ 45 ∧ 38 ∧ 23 ∧ 44 ∧ 20 ⟶ 49
3 ∧ 24 ∧ 49 ∧ 31 ∧ 34 ∧ 19 ∧ 8 ∧ 13 ⟶55
204 ∧ 222 ∧ 112 ∧ 66 ∧ 155 ⟶ 269
41 ∧ 42 ∧ 5 ∧ 8 ∧ 40 ∧ 25 ∧ 16 ∧ 36 ⟶49
17 ∧ 199 ∧ 84 ∧ 5 ∧ 176 ⟶ 262
183 ∧ 56 ∧ 145 ∧ 141 ∧ 148 ∧ 92 ∧ 175 ⟶ 186
82 ∧ 114 ∧ 24 ⟶ 216
14 ∧ 96 ∧ 18 ∧ 49 ⟶ 98
60 ∧ 102 ∧ 20 ∧ 29 ∧ 66 ∧ 86 ∧ 96 ∧ 44 ⟶186
17 ∧ 87 ∧ 42 ∧ 91 ∧ 46 ∧ 23 ∧ 48 ⟶ 101
3 ∧ 1 ⟶ 6
39 ∧ 33 ∧ 9 ∧ 26 ∧ 61 ∧ 12 ∧ 6 ⟶ 65
155 ⟶ 246
34 ∧ 33 ∧ 25 ∧ 22 ∧ 21 ∧ 6 ⟶ 39
79 ∧ 95 ∧ 216 ∧ 199 ∧ 40 ∧ 18 ∧ 238 ∧ 15 ⟶277
55 ∧ 17 ∧ 46 ⟶ 77
24 ∧ 9 ∧ 8 ∧ 190 ∧ 172 ⟶ 237
2 ∧ 131 ∧ 54 ⟶ 146

37 ∧ 17 ∧ 21 ∧ 48 ∧ 146 ∧ 24 ∧ 36 ∧ 56 ⟶155
187 ∧ 206 ∧ 47 ∧ 146 ⟶ 296
95 ∧ 111 ∧ 77 ∧ 210 ∧ 166 ⟶ 293
8 ∧ 48 ∧ 37 ∧ 22 ∧ 29 ⟶ 67
235 ∧ 44 ∧ 70 ∧ 131 ∧ 27 ∧ 190 ∧ 55 ⟶ 240
41 ∧ 207 ∧ 197 ⟶ 216
3 ∧ 187 ∧ 125 ∧ 135 ∧ 48 ∧ 55 ⟶ 234
58 ∧ 135 ∧ 89 ∧ 170 ∧ 73 ∧ 223 ∧ 68 ⟶ 224
131 ∧ 81 ∧ 2 ∧ 142 ∧ 86 ⟶ 217
6 ⟶ 55
5 ∧ 2 ∧ 10 ∧ 8 ∧ 7 ∧ 9 ∧ 3 ∧ 11 ⟶14
7 ∧ 100 ∧ 159 ∧ 45 ∧ 158 ∧ 99 ⟶ 187
1 ∧ 65 ⟶ 66
101 ∧ 81 ∧ 67 ∧ 139 ∧ 94 ∧ 95 ⟶ 269
77 ∧ 176 ∧ 217 ∧ 34 ∧ 35 ⟶ 245
3 ⟶ 4
178 ∧ 239 ∧ 158 ∧ 52 ∧ 209 ∧ 123 ⟶ 253
63 ∧ 90 ⟶ 96
121 ∧ 39 ∧ 117 ∧ 118 ∧ 7 ∧ 43 ∧ 69 ∧ 92 ⟶166
69 ∧ 116 ∧ 128 ∧ 17 ⟶ 146
50 ∧ 3 ⟶ 55
16 ∧ 8 ∧ 48 ∧ 7 ∧ 11 ∧ 50 ∧ 12 ⟶ 65
160 ∧ 25 ∧ 140 ∧ 50 ∧ 187 ∧ 59 ∧ 157 ⟶ 188
114 ∧ 91 ⟶ 146
54 ∧ 6 ⟶ 171
5 ⟶ 96
79 ∧ 213 ∧ 268 ∧ 129 ∧ 56 ⟶ 270
70 ⟶ 92
186 ∧ 113 ⟶ 200
2 ∧ 32 ∧ 29 ∧ 40 ∧ 22 ∧ 53 ⟶ 65
68 ∧ 108 ∧ 96 ⟶ 199
35 ∧ 149 ∧ 110 ∧ 144 ∧ 24 ∧ 218 ∧ 221 ⟶ 276
182 ∧ 29 ∧ 247 ∧ 160 ∧ 162 ∧ 113 ∧ 86 ∧ 26 ⟶251
143 ∧ 86 ∧ 9 ∧ 81 ⟶ 230
38 ⟶ 111
86 ∧ 6 ∧ 97 ∧ 157 ∧ 105 ⟶ 221
59 ∧ 67 ∧ 38 ∧ 46 ∧ 47 ∧ 39 ∧ 19 ∧ 13 ⟶113
18 ∧ 32 ∧ 15 ∧ 11 ∧ 31 ∧ 10 ∧ 9 ⟶ 33
99 ∧ 116 ⟶ 226
143 ∧ 167 ∧ 249 ∧ 243 ∧ 161 ∧ 260 ∧ 15 ⟶ 279
35 ∧ 47 ∧ 1 ∧ 14 ∧ 64 ∧ 15 ∧ 26 ∧ 69 ⟶85
133 ∧ 186 ∧ 104 ∧ 125 ∧ 97 ⟶ 200
20 ∧ 151 ∧ 14 ⟶ 176
2 ∧ 55 ∧ 84 ∧ 92 ∧ 5 ∧ 96 ⟶ 98

112 ∧ 74 ∧ 17 ∧ 137 ∧ 150 ∧ 73 ∧ 11 ∧ 6 ⟶159
116 ∧ 25 ∧ 88 ∧ 18 ⟶ 119
48 ∧ 134 ∧ 132 ∧ 80 ∧ 106 ∧ 30 ∧ 150 ⟶ 153
5 ∧ 1 ∧ 10 ⟶ 31
161 ∧ 234 ∧ 92 ⟶ 237
140 ∧ 30 ∧ 98 ∧ 154 ∧ 183 ∧ 123 ∧ 86 ∧ 50 ⟶188
56 ∧ 1 ⟶ 92
56 ⟶ 148
25 ∧ 189 ∧ 148 ∧ 93 ∧ 150 ∧ 57 ∧ 110 ⟶ 224
17 ∧ 4 ∧ 15 ⟶ 39
30 ∧ 5 ∧ 7 ∧ 19 ⟶ 45
72 ∧ 71 ∧ 112 ∧ 63 ⟶ 206
51 ∧ 106 ∧ 30 ∧ 47 ∧ 28 ∧ 67 ∧ 39 ∧ 58 ⟶112
106 ∧ 1 ∧ 271 ∧ 125 ∧ 178 ∧ 144 ∧ 111 ∧ 165 ⟶299
6 ∧ 12 ∧ 75 ∧ 48 ∧ 64 ∧ 33 ⟶ 97
284 ∧ 35 ∧ 116 ∧ 30 ∧ 194 ∧ 252 ∧ 82 ∧ 17 ⟶292
122 ∧ 26 ∧ 65 ∧ 3 ∧ 9 ∧ 44 ∧ 113 ∧ 133 ⟶157
86 ∧ 141 ∧ 144 ∧ 23 ∧ 47 ∧ 48 ∧ 114 ∧ 87 ⟶147
34 ∧ 6 ∧ 9 ∧ 41 ∧ 17 ∧ 18 ∧ 24 ⟶ 65
274 ∧ 260 ∧ 115 ⟶ 293
19 ∧ 80 ⟶ 161
127 ∧ 136 ∧ 171 ∧ 18 ∧ 82 ∧ 48 ⟶ 187
13 ∧ 6 ∧ 223 ∧ 59 ∧ 149 ⟶ 245
130 ⟶ 143
101 ∧ 39 ∧ 177 ∧ 58 ∧ 165 ⟶ 211
33 ∧ 124 ∧ 36 ∧ 8 ∧ 117 ⟶ 199
65 ∧ 86 ∧ 195 ⟶ 215
151 ∧ 24 ⟶ 176
161 ∧ 10 ∧ 18 ⟶ 246
17 ∧ 25 ∧ 22 ∧ 43 ∧ 9 ∧ 53 ⟶ 69
106 ∧ 167 ∧ 39 ∧ 84 ∧ 127 ⟶ 187
97 ∧ 124 ∧ 102 ∧ 128 ∧ 183 ∧ 149 ∧ 180 ⟶ 187
175 ∧ 240 ∧ 167 ∧ 52 ∧ 160 ⟶ 251
46 ∧ 37 ∧ 17 ∧ 40 ∧ 5 ∧ 48 ∧ 2 ⟶ 111
64 ∧ 76 ∧ 128 ∧ 129 ∧ 100 ⟶ 157
20 ∧ 165 ∧ 219 ∧ 241 ∧ 191 ∧ 61 ⟶ 251
26 ∧ 17 ∧ 16 ⟶ 34
113 ∧ 119 ∧ 45 ∧ 12 ∧ 39 ∧ 213 ∧ 68 ∧ 99 ⟶220
208 ∧ 174 ∧ 189 ∧ 207 ∧ 89 ∧ 105 ∧ 232 ⟶ 246
16 ∧ 72 ∧ 71 ∧ 108 ⟶ 144
96 ∧ 84 ∧ 44 ∧ 93 ⟶ 108
117 ∧ 142 ⟶ 237
202 ∧ 123 ∧ 10 ∧ 68 ⟶ 220
181 ∧ 200 ∧ 15 ∧ 21 ∧ 194 ∧ 65 ⟶ 221

$47 \wedge 123 \wedge 55 \wedge 71 \longrightarrow 173$
$87 \wedge 79 \longrightarrow 102$
$79 \wedge 121 \wedge 194 \longrightarrow 245$
$113 \longrightarrow 279$
$156 \wedge 35 \wedge 33 \wedge 37 \wedge 245 \wedge 135 \wedge 233 \longrightarrow 247$
$77 \wedge 85 \wedge 35 \longrightarrow 110$
$51 \wedge 13 \wedge 49 \wedge 76 \wedge 60 \wedge 9 \longrightarrow 135$
$40 \wedge 95 \wedge 221 \wedge 4 \wedge 26 \wedge 217 \wedge 151 \wedge 55 \longrightarrow 227$
$146 \wedge 48 \wedge 138 \wedge 67 \longrightarrow 156$
$88 \wedge 14 \wedge 49 \wedge 9 \wedge 13 \longrightarrow 134$
$169 \wedge 113 \wedge 43 \wedge 21 \wedge 53 \wedge 110 \longrightarrow 219$
$78 \wedge 123 \wedge 37 \wedge 7 \wedge 14 \wedge 29 \wedge 121 \wedge 75 \longrightarrow 157$
$52 \longrightarrow 135$
$23 \wedge 111 \wedge 75 \wedge 140 \wedge 4 \wedge 117 \wedge 102 \longrightarrow 144$
$89 \wedge 177 \longrightarrow 224$
$5 \wedge 11 \wedge 98 \longrightarrow 174$
$20 \wedge 19 \wedge 166 \wedge 131 \wedge 182 \wedge 63 \longrightarrow 211$
$52 \longrightarrow 90$
$54 \wedge 57 \wedge 59 \wedge 64 \wedge 51 \wedge 21 \longrightarrow 97$
$12 \wedge 73 \wedge 138 \wedge 54 \wedge 21 \wedge 14 \wedge 151 \longrightarrow 155$
$139 \wedge 14 \wedge 232 \wedge 11 \wedge 98 \wedge 238 \wedge 121 \wedge 52 \longrightarrow 271$
$14 \wedge 36 \wedge 39 \wedge 43 \wedge 69 \wedge 85 \wedge 122 \wedge 171 \wedge 188 \wedge 199 \wedge 200 \wedge 212 \wedge 214 \wedge 224 \wedge 226 \wedge$ $229 \wedge 230 \wedge 245 \wedge 247 \wedge 251 \wedge 253 \wedge 262 \wedge 264 \wedge 270 \wedge 294 \wedge 296 \longrightarrow 300$